A Lawyer's Guide to Modern Valuation Techniques in Mergers and Acquisitions

Samuel C. Thompson Jr.

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A Lawyer's Guide to Modern Valuation Techniques in Mergers and Acquisitions

Samuel C. Thompson, Jr.*

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I. INTRODUCTION

The starting point for both the buyer and the seller in any merger or acquisition transaction is to determine the value of the target corporation. For the buyer (i.e., the acquiring corporation), this is a capital budgeting decision similar to any other. invest-

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2. J. FRED WESTON ET AL., Mergers Restructuring and Corporate Control 132 (1990) [hereina-
ment decision, such as the decision to build a new plant. For the seller (i.e., either the target corporation in a sale of assets or its shareholders in a sale of stock), the determination of the target's value sets the reservation price at which the seller will stop holding and sell.

There are many techniques for determining the value of the assets or shares of a target corporation. These include: (1) valuation based on comparable target corporations—a similar to the traditional technique for valuing real estate; (2) valuation based on comparable transactions—looking to transactions that are similar to the one in which the stock or assets of the target are being acquired; (3) valuation based on the liquidation value of the assets of the target corporation; (4) valuation based on the replacement value of the target's assets; (5) valuation based on a leveraged buyout (LBO) analysis by a financial buyer; and (6) valuation based on the discounting to present value of the target's expected future cash flows through the use of the discounted cash flow (DCF) technique, with the discount rate determined by the use of (a) the capital asset pricing model (CAPM), (b) arbitrage pricing theory (APT), or (c) the weighted average cost of capital (WACC). This article focuses on DCF, CAPM, APT, and WACC, modern valuation techniques based on economic models of valuation. The DCF model provides the mechanism for discounting to present value the target's expected future cash flows, with the discount rate provided by either the CAPM model, the APT model, or the WACC model.

Modern finance theory teaches that the value of an investment, such as a target corporation, is not determined by accounting conventions, but rather equals the present value of the cash flows expected to be produced by the investment, discounted at a rate that properly reflects the risk associated with the investment. Thus, expected net cash

after WESTON ET AL., MERGERS] (explaining that “capital budgeting is a form of cost-benefit analysis, a method of determining whether benefits exceed costs when both are properly measured and evaluated”).

3. This technique is commonly known as “the comparable company technique.”

4. This technique is commonly known as “the comparable transaction technique.”

5. A strategic buyer intends to operate the target’s assets for the long term. By contrast, a financial buyer intends to buy the target and then dispose of its assets in the short term at a profit.


7. See, e.g., Copeland ET AL., Valuation, supra note 6, at 75.

8. See, e.g., BreaLEY & Myers, supra note 6, ch. 2; Shapiro, supra note 6, ch. 2. See generally Steve
flow, and not reported financial earnings, is one of the key elements in modern valuation techniques. The DCF, CAPM, APT, and WACC models are the principal tools for implementing this modern theory. These models are regularly employed by investment bankers and other valuation specialists involved in mergers and acquisitions.

In many situations it may be appropriate to utilize multiple techniques in valuing a target. For that reason, references here are also made to the comparable transactions and comparable companies’ techniques, which are also often used by valuation specialists. The option valuation approach, which may be used when future investment decisions are not known at the inception of a project, is not examined here.

This article is written for lawyers, judges, and other non-valuation specialists who may be involved in mergers, acquisitions, and related transactions through procedures such as appraisal and fairness proceedings. Although such professionals are not required to be experts in valuation techniques, in many instances they need a basic understanding of the DCF, CAPM, APT, and WACC models.

In their treatise on corporate finance, Professors Brealey and Myers emphasize the importance of the DCF method of valuation, stating that “[v]alue today always equals future cash flow discounted at the opportunity cost of capital.” Brealey and Myers explain that the opportunity cost of capital—the discount or “hurdle” rate—is the “return foregone by investing in the [target] rather than securities.” According to Brealey and Myers, CAPM is the “best-known model of risk and return” and is preferred over the APT model, due to the APT model’s elaborateness and complexity, for determining the discount rate to be used within the DCF model.

Both CAPM and APT are derived from the efficient capital markets hypothesis, which asserts that if information is easily available to investors in a securities market, security prices will reflect all relevant and ascertainable information. However, a 1992 empirical study of CAPM by Professors Fama and French has called into question the validity of the hypothesis. Their findings have been challenged by Professors Roll and Ross. In any event, it seems likely that financial analysts and investment bankers


9. See, e.g., Brian H. Saffer, Touching All Bases in Setting Merger Prices, MERGERS AND ACQUISITIONS 42 (Fall 1984).

10. See, e.g., SHAPIRO, supra note 6, at 223-30.

11. See, e.g., Hanson Trust PLC v. MLSCM Acquisition Inc., 781 F.2d 264, 275 (2d Cir. 1986) (noting that in evaluating a hostile tender offer the target’s board was obligated to become familiar with the investment banker’s report).

12. BREALEY & MYERS, supra note 6, at 63.

13. Id. at 13.

14. Id.

15. Id. at 174.

16. Id. (explaining that the APT model “offers an alternative theory of risk and return”).


19. Richard Roll & Stephen A. Ross, On the Cross-Sectional Relation Between Expected Returns and
will continue to use CAPM. The model enjoys prominence in the leading texts on corporate finance and has been taught to thousands of MBA students.

The Fama and French critique of CAPM, discussed below, may both increase the use of APT in determining discount rates and possibly decrease the importance of the DCF model. The model has no utility without the use of an appropriate discount rate. On the other hand, as demonstrated in the discussion of APT, in many instances the discount rate determined using CAPM approximates the rate determined using APT. Thus, although CAPM may not produce the precise cost of capitals, it should produce a rate that is a close approximation. Since valuation is an inexact science, it would not be appropriate at this time to completely abandon CAPM.

Part II of this Article outlines various types of acquisitions and other contexts in which a knowledge of modern valuation concepts can be important to persons who are not valuation experts. Part III introduces the concepts of present value and net present value (NPV), which are at the heart of the DCF technique. Part III also discusses the related concept of internal rate of return (IRR). Part IV expands on the DCF and NPV techniques in situations involving multiple cash flows. Part V provides guidelines for estimating the free cash flows and terminal value used in the DCF technique. Part VI explores the determination, through the use of CAPM, of the discount rate used in the DCF model. Part VI.G discusses the recent critique of CAPM by Professors Fama and French.

Part VII discusses the use of the APT model in determining the discount rate used in the DCF model. This APT model is based on multiple factors, many of which are difficult to identify and quantify, whereas CAPM is based on three factors, each of which generally can be easily determined. Part VIII examines the interaction between the investment decision and the financing decision. Part IX discusses the weighted average cost of capital (WACC) approach, which in appropriate circumstances may be used in lieu of CAPM or APT for determining the discount rate to be used in the DCF model.

Part X discusses the use of DCF in acquisitions that produce synergies. Synergies arise when the combination of the acquiror and the target results in reduced costs or increased revenue that could not be realized if the two firms remained independent. Part XI illustrates the application of DCF and CAPM in determining the price of a target in a merger. Part XII provides an introduction to non-DCF valuation techniques. Finally, Part XIII contains the conclusion.

The Appendices contain excerpts from various documents that were prepared in connection with Time, Inc.'s acquisition of Warner Communications, Inc. These documents relate to the valuation of Warner by the board of directors of both Warner, the target, and Time, the acquiror. In addition, Appendix F contains excerpts from the proxy statement involving the recent acquisition by Bank America of Security Pacific.
II. IMPORTANCE OF MODERN VALUATION CONCEPTS FOR NON-VALUATION EXPERTS

A. Impact in Appraisal Proceedings Under Delaware Law

The Delaware Supreme Court's decision in *Weinberger v. UOP, Inc.* highlights the importance of modern valuation concepts. In *Weinberger*, a controlling corporate shareholder of the target acquired the target in a freeze-out merger. The Delaware Supreme Court found that the controlling shareholder breached its fairness duty to the public shareholders. Although the court granted recovery to the public shareholders in their challenge to the merger, the court held that for subsequent proceedings the remedy for complaining shareholders is to formally dissent from the transaction and to have their shares appraised. The court further held that in determining the value of stock in appraisal proceedings, Delaware courts are no longer required to use the Delaware Block Method exclusively, but may follow a more "liberal approach." This liberal approach is to include "proof of value by any techniques or methods which are generally considered acceptable in the financial community." In *Weinberger*, the plaintiff's investment analyst used both the DCF technique and the comparable transactions approach in valuing the target company.

Under the Delaware Block Method, the value of a firm is determined by taking a weighted average of the following four factors:

1. the market or trading value of the target's shares before the merger;
2. the value of the target's shares determined by multiplying its average recent earnings by an appropriate multiple (i.e., capitalization of earnings);
3. the value of the target determined by considering the net asset value of the target (i.e., fair market value of assets less liabilities); and
4. the value of the target based on its past dividend stream.

The weights for the four factors are determined by the particular facts and circumstances, and in appropriate cases, a factor may have a weight of zero.

The Delaware Block Method has been followed by other states; however, in the view of the authors of a leading casebook on corporations, "no professional analyst would deem it acceptable." These same authors say that although some Delaware
cases have utilized the DCF technique," it appears that “no new standard practice has developed in Delaware.” As noted below, however, one Delaware case has described the DCF technique as “prominent.”

The American Law Institute’s Principles of Corporate Governance project proposes that in appraisal proceedings involving arm’s length transactions, “fair value should be determined using the customary valuation concepts and techniques generally employed in the relevant securities and financial markets for similar businesses in the context of the transaction giving rise to appraisal.” The American Law Institute (ALI) explains that this approach follows “prevailing Delaware law in mandating that the court use the relevant financial valuation techniques generally employed in the financial community.” The ALI goes on to explain that “few subsequent decisions have addressed the criteria that are to be employed in the wake of Delaware’s rejection of the old ‘block’ formula.” The Reporter’s Notes to this provision further explain:

Since Weinberger, the Delaware courts have increasingly turned to a discounted cash flow technique, under which an appraisal court makes three distinct computations: First, it estimates the net cash flow that the firm will generate over some foreseeable period; second, it determines a terminal or residual value as of the end of this first period, which represents the value of the firm’s expected cash flows thereafter; and finally, it determines the appropriate cost of capital by which to discount to a present value both the projected future cash flows and the estimated terminal or residual value.

In the 1991 appraisal proceeding in In re Radiology Associates, Inc. Litigation, the Delaware Chancery Court accepted (with adjustments) the DCF valuation approach proposed by the plaintiff’s valuation expert and rejected the Delaware Block Method, which was proposed by the defendant’s valuation expert. The court explained that “[e]ven though the Delaware courts have used the Delaware Block Method infrequently since Weinberger, the Delaware courts still consider it an acceptable procedure for valu-

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33. CHOPER ET AL., CORPORATIONS, supra note 31, at 1184.
35. AMERICAN LAW INSTITUTE, PRINCIPLES OF CORPORATE GOVERNANCE: ANALYSIS AND RECOMMENDATIONS § 7.22(a) (The American Law Institute 1992) [hereinafter ALI PROJECT].
36. Id. at 315-16 cmt. a.
37. See, e.g., infra part V.A-H (discussing estimation of free cash flows).
38. See, e.g., infra part V.I (discussing estimation of terminal value).
39. See, e.g., infra parts VI-VII and IX (discussing CAPM, APT, and WACC).
40. See, e.g., infra parts III-IV (discussing PV, NPV, IRR, and DCF).
42. 611 A.2d 485 (Del. Ch. 1991).
ing a company." The court, however, gave "no weight" to the asset value, market value, or earnings value used by the defendant's expert in applying the Delaware Block Method.\(^{44}\)

The court in *Radiology Associates* found that the DCF technique used by the plaintiff's expert was basically sound.\(^{45}\) The projected revenues and terminal value were found to be objectively based and reasonable after adjustment by the court, and the discount rate was appropriately determined through the use of both CAPM and WACC.\(^{46}\)

The court in *Radiology Associates* rejected use by the plaintiff's expert of the comparable companies methodology because of a failure to identify a group of comparable companies.\(^{47}\) The court explained that under this approach, after the first step of identifying comparable public companies the "approach calculates the value of the company through the use of earnings and other multiples."\(^{48}\)

In a similar appraisal proceeding in *Cede & Co. v. Technicolor, Inc.*,\(^{49}\) the Delaware Chancery Court performed a detailed analysis of the DCF techniques employed by the experts for each side. In introducing the DCF concept the court stated:

In many situations, the discounted cash flow technique is in theory the single best technique to estimate the value of an economic asset. Prior to our Supreme Court's decision in *Weinberger . . .*, however, that technique was not typically employed in appraisal cases in this jurisdiction. But with *Weinberger*'s implicit encouragement, this technique has become prominent.\(^{50}\)

The *Technicolor* court also gave the following general description of the DCF technique:

The DCF model entails three basic components: an estimation of net cash flows that the firm will generate and, when, over some period; a terminal or residual value equal to the future value, as of the end of the projection period, of the firm's cash flows beyond the projection period; and finally a cost of capital with which to discount to a present value both the projected net cash flows and the estimated terminal or residual value.\(^{51}\)

Although both of the experts in the *Technicolor* case\(^{52}\) used the DCF technique, the court said that the experts' opinions of the value "cover[ed] an astonishing

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43. *Id.* at 496.
44. *Id.* at 498; *see also* Harris v. Rapid-American Corp, No. CIV. 6462, 1990 LEXIS 166, (Del. Ch. Oct. 2, 1990) (approving the use of the comparable companies method).
46. *Id.* at 492-93.
47. *Id.* at 489-90.
48. *Id.* at 489. The use of the comparable companies method was approved by the court in Harris v. Rapid-American Corp, No. CIV. 6462, 1990 LEXIS 166, (Del. Ch. Oct. 2, 1990).
50. *Id.* at *7*.
51. *Id.*
52. One of whom was Professor Rappaport, whose book, *CREATING SHAREHOLDER VALUE*, is cited throughout this article.
The court explained: "Two experts looking at the same historic data and each employing a discounted cash flow valuation technique arrive at best estimates as different as $13.14 per share and $62.75 per share."

The Technicolor court correctly explains why it is possible to get such a difference in valuations using the DCF technique:

While the basic three-part structure of the two DCF models of the same firm, as of the same date, will be the same, it is probably the case (and is certainly true here) that the details of the analysis may be quite different. That is, not only will assumptions about the future differ, but different methods may be used within the model to generate inputs.

B. Impact in Fairness Opinions and in SEC Disclosure Documents

The DCF technique is likely to be employed by investment bankers as one of the bases for the issuance of fairness opinions in a variety of contexts, including mergers, tender offers, and leveraged buyouts. Although investment bankers have no established industry guidelines for rendering fairness opinions, "[a]lmost without exception" they use some form of DCF in conjunction with an analysis of comparable transactions, comparable companies, and liquidation value. Both Lazard and Wasserstein/Shearson used, among other things, DCF, comparable companies, and comparable transactions in valuing Warner.

As an example of a fairness opinion, Appendix B contains excerpts from the opinion issued by Lazard to the board of directors of Warner. The opinion does not indicate the methodology utilized in preparing the valuation, but merely states the factual back-

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54. Id.
55. Id. at *8.
56. See, e.g., Lucian Arye Babchuk & Marcel Kahan, *Fairness Opinions: How Fair Are They and What Can Be Done About It?,* 1989 DUKE L.J. 27 (1989). This article briefly describes the DCF and CAPM techniques. Id. at 34-35. Other valuation techniques mentioned in the article are: (1) the value of the firm's net assets; (2) a multiple of the firm's past earnings; (3) the discounted value of future dividends; (4) the market price of the shares; and (5) an average of these measures. Id. at 34-35; see also William J. Carney, *Fairness Opinions: How Fair Are They and Why We Should Do Nothing*, 70 WASH. U. L.Q. 523 (1992) (arguing for no liability except in cases of fraud); Leonard Chazen, *Fairness from a Financial Point of View in Acquisitions of Public Companies: Is “Third-Party Sale Value” the Appropriate Standard?*, 36 BUS. LAW. 1439 (1981); Charles M. Elson, *Fairness Opinions: Are They Fair or Should We Care?*, 53 OHIO ST. L.J. 951 (1992); Ted J. Fiflis, *Responsibility of Investment Bankers to Shareholders*, 70 WASH. U. L.Q. 497 (1992) (proposing gatekeeper liability); Robert J. Giuffra, Jr., *Note, Investment Banker’s Fairness Opinions in Corporate Control Transactions*, 96 YALE L.J. 119, 132 (1986) (arguing that when directors rely on fairness opinions they should be obligated “(1) to select the investment banker with care, (2) to disclose accurate information to the investment banker, (3) to determine whether the investment banker followed accepted valuation procedures, and (4) to examine the investment banker’s conclusions”) [hereinafter *Investment Banker’s Fairness Opinions*]; Dale A. Osterlie, *Fairness Opinions as Magic Pieces of Paper*, 70 WASH. U. L.Q. 541 (1992) (arguing that the legal relationship between the investment banker and the firm should be governed by fiduciary principles rather than straight contract law).
57. *Investment Banker’s Fairness Opinions*, supra note 56, at 137.
58. Id. at 137-38.
ground and expresses the opinion of Lazard that the merger consideration is fair to the Warner shareholders from a financial viewpoint. Appendix D, which contains excerpts from Lazard’s valuation study, illustrates how Lazard utilized the DCF technique as one of the bases for rendering its fairness opinion.

Fairness opinions have become particularly important in mergers and acquisitions in view of the Delaware Supreme Court’s opinion in Smith v. Van Gorkom. The practical effect of the Van Gorkom decision is to force publicly held firms to seek fairness opinions from investment bankers in merger, acquisition, and similar corporate change transactions. Further, the Second Circuit in Hanson Trust PLC v. MLSCM Acquisition Inc. held that in order for the business judgment rule to apply in the context of a hostile tender offer, the target’s board was obligated to become familiar with the investment banker’s report. The importance of the fairness opinions in establishing the due care taken by the boards of Time and Warner is illustrated in the discussions of the opinions in the Time and Warner Joint Information Statement and Time Prospectus, which is attached as Appendix A.

Although a fairness opinion is not required in a going private transaction governed by SEC Rule 13E-3, that rule requires the parties to indicate whether the transaction is fair or unfair to the target’s shareholders and whether a fairness opinion was received in the transaction. If a fairness opinion is utilized in a going private transaction, Item 9(b) of Schedule 13E-3 requires a detailed description of the opinion including the procedures followed in preparing the opinion and the “bases for and methods of arriving at [the] findings or recommendations.” Thus, this provision requires disclosure of the information in the “blue books” prepared by the investment bankers for presentation to the boards of directors.

The excerpt in Appendix A from the Time and Warner Joint Information Statement and Time Prospectus illustrates this type of disclosure in the context of the final going private merger in which Time completed its acquisition of Warner. This transaction was governed by Rule 13e-3 and Schedule 13E-3. Appendices C and D contain excerpts from the blue books of the investment bankers for Time and Warner. These blue books were required to be filed with the SEC as part of Schedule 13E-3.

A similar type of comprehensive disclosure is also sometimes required now by the SEC staff in arm’s length acquisitions that are not going private transactions. Both Item

59. 488 A.2d 858 (Del. 1985). In Van Gorkom the court held that the directors of the target corporation violated their duty of care in agreeing to have the target acquired in a friendly merger. The court found that the target’s directors had not adequately informed themselves as to the value of the target, having relied principally on the chairman for the valuation. One factor which influenced the court’s decision was the directors’ failure to follow the normal (although not legally required) practice of obtaining an opinion of an investment banker to the effect that the transaction was fair to the target’s shareholders.


61. 781 F.2d 264, 275 (2d Cir. 1986).

62. A going private transaction consists of certain acquisitions of publicly held corporations by controlling shareholders.


64. The “blue book” contains the analysis which serves as the basis of the fairness opinion.
14(a)(10) of Schedule 14A and Item 4(b) of Form S-4 provide that if a fairness opinion materially relating to an arm's length acquisition subject to those provisions has been received, and the opinion is referred to in the prospectus or proxy statement, then the parties are required to furnish the same information that would be required by Item 9(b) of Schedule 13E-3 under the going private rules. Thus, the same detailed disclosures concerning fairness opinions and blue books are required in both going private and arm's length acquisitions of publicly held firms. There is no requirement, however, to file the blue books in arm's length transactions.

Notwithstanding this uniformity in the disclosure requirements, the SEC staff has previously permitted summary disclosures regarding fairness opinions and blue books in arm's length transactions. This policy has recently changed, at least in significant arm's length transactions, and the staff now generally requires detailed disclosure of the analyses on which any fairness opinion is based. Appendix E, which is an excerpt from the Bank America and Security Pacific Joint Proxy Statement and Bank America Prospectus, illustrates this type of complete disclosure in the arm's length acquisition of Security Pacific by Bank America. As indicated in Appendix E, Bank America's investment banker used, inter alia, the comparable company, the comparable transaction, and the DCF techniques. Obviously, it is incumbent on those involved in the preparation of SEC disclosure documents in which fairness opinions are discussed to be generally familiar with the valuation techniques utilized.

C. Impact in Other Contexts

DCF, CAPM, APT, and WACC may also be employed in determining whether a fraudulent transfer has occurred in a leveraged buyout (LBO) or similar transaction. In such situations, the secured creditors provide new debt to fund the LBO, and unless the target's assets have a sufficient value, the transaction may be viewed as operating as a fraud on the unsecured creditors. The issue is whether the LBO firm is solvent after the transaction. If the firm is insolvent, a fraudulent transfer has occurred.

Courts use two basic valuation standards in determining whether a firm is solvent: (1) the liquidation method; and (2) the going concern method. One writer has argued that the going concern method is the appropriate standard and that the DCF technique should be used in determining going concern value.

In LBOs and similar transactions, it is common for the lenders to receive from an appraisal company an opinion that the target will be solvent after the acquisition. Appendix E contains an excerpt from the solvency opinion that was issued to the lenders

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65. Schedule 14A contains the SEC proxy rules.
66. Form S-4 is the SEC's business combination registration statement.
68. Id.
70. Id. at 384.
71. Id. at 422-25.
72. Id. at 426-28.
in Time's acquisition of Warner. The opinion defines the "fair salable value" of the resulting company as "the aggregate amount at which the assets of the company, valued in its entirety as a going concern, would change hands in the open market." It appears from the opinion that the DCF model was one of the techniques employed.73

Finally, DCF, CAPM, APT, and WACC may be used in other valuation contexts, such as determining the offering price of shares in an initial public offering and determining the division of new securities issued in a bankruptcy reorganization. CAPM, APT, and WACC may also be utilized in utility rate-making cases to determine the rate of return a regulated utility should be allowed to earn.74 In summary, although this Article focuses on the use of these modern techniques in the valuation of acquisition targets, these concepts have significance in a variety of contexts.

III. INTRODUCTION TO PRESENT VALUE, NET PRESENT VALUE, AND INTERNAL RATE OF RETURN75

A. The Basic Concept of Present Value (PV)

One of the basic principles of finance is that a dollar received today is worth more than a dollar received tomorrow.76 This is because a dollar received today can be invested today and begin earning an immediate return, whereas a dollar received tomorrow cannot be invested until tomorrow. This fundamental principle is at the heart of the present value concept: The present value of one dollar to be received at some point in the future is less than one dollar. To calculate the present value of a future amount the future amount must be multiplied by a discount factor. The discount factor is naturally less than one. If it were equal to one the implication would be that a dollar today is worth the same as a dollar tomorrow, and if it were greater than one the implication would be that a dollar tomorrow is worth more than a dollar today.

The present value concept can be expressed algebraically by the following formula, where PV means present value and FA means the future amount:

\[
PV = (\text{Discount Factor}) \times (FA_n)
\]

The \(n\) represents the particular period during which the FA is to be received. The discount factor is a fraction that is the reciprocal of one plus the required rate of return, raised to the power of \(n\). This required rate of return, which is also known as the "discount rate," the "hurdle rate," or the "cost of capital," is represented by \(r\). The discount factor is written as follows:

\[
\text{Discount Factor} = \frac{1}{(1 + r)^n}
\]

73. See infra Appendix E.
74. See, e.g., INVESTMENTS, supra note 6, at 242 (providing for use of CAPM in utility rate-making cases).
75. See BREALEY & MYERS, supra note 6, ch. 2; SHAPIRO, supra note 6, ch. 2.
76. BREALEY & MYERS, supra note 6, at 12.
The discount rate \( (r) \) is the reward that investors demand for making an investment today, and \( n \) represents the number of periods (e.g., years) that elapse before the particular payment (i.e., FA) is received. The discount factor can be obtained from standard charts which generally have on the vertical axis the number of years which must run until the payment is received (i.e., \( n \)) and on the horizontal axis the discount rate (i.e., \( r \)). These charts are generally provided as appendices to texts about corporate finance and are similar to the following:

<table>
<thead>
<tr>
<th>Number of Years ((n))</th>
<th>Interest Rate Per Year ((r))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>.990</td>
</tr>
<tr>
<td>2</td>
<td>.980</td>
</tr>
<tr>
<td>3</td>
<td>.971</td>
</tr>
<tr>
<td>5</td>
<td>.951</td>
</tr>
<tr>
<td>10</td>
<td>.905</td>
</tr>
</tbody>
</table>

The greater the number of years to run before the FA is to be received and the higher the interest rate, the lower the discount factor and the present value of the payment.

The greater the risk associated with an investment, the greater the reward investors will demand for making the investment. The present value of a future payment of a safe dollar is worth more than the present value of the future payment of a risky dollar. Therefore, the greater the risk, the greater the required return. One of the purposes of CAPM, APT, and WACC is to determine the appropriate discount rate given the particular level of risk associated with the investment.

These concepts are illustrated as follows:

Example (1). Assume that acquiring corporation (AC) is considering the acquisition of all the stock of a closely held target corporation (TC) and AC wants to determine how much it should pay for the stock of TC. Assume further that AC is confident that after operating TC for one year it can sell all the stock of TC for $1 million, and that AC intends to make such a sale. Consequently, in this situation \( n \) equals one year because that is when the FA will be paid. Also, assume that because TC is in a growth business it will not be paying any dividends during the year that AC plans to hold its stock. Thus, there will be only one FA of $1 million. Finally, assume that AC has determined (by using CAPM, APT, or WACC) that given the risk associated with an investment in TC, the required discount rate is 15% (i.e.,...
Modern Valuation Techniques in Mergers and Acquisitions

$r = 15\%$). In such case the present value of the stock of TC can be computed as follows (with the discount factor obtained from the above chart):

$$PV = \frac{1}{1 + r} \times (FA)$$

$$PV = \frac{1}{1 + .15} \times ($1\ million)$$

$$PV = .87 \times ($1\ million)$$

$$PV = $870,000$$

The present value of the expected $1 million payment on the sale of TC in one year is $870,000. Thus, as long as AC can purchase the stock of TC for no more than $870,000, AC will not have overpaid for TC.

The above example shows that there are two variables in measuring present value: (1) an estimate of the FA; and (2) the determination of $r$, the discount rate. The higher the estimate of the FA, the higher the PV will be, but the higher the $r$, the lower the PV.

There is just one FA in the above example; however, as a practical matter in most acquisition transactions it can be expected that there will be periodic FAs. An investment in a target can be expected to produce a current return (e.g., dividends or free cash flow) and an expected terminal FA, which is an estimate of the final sale or liquidation value of the target. In order for the present value formula to work there must be either an estimate of the terminal FA or an infinite projection of free cash flows.81

The present value can be computed on a calculator that has the capacity to compute present values by the following three-step process: (1) enter each expected future payment (FA) for the period ($n$) during which such payment is expected; (2) enter the required rate of return ($r$); (3) hit the PV button to determine the present value of such payments.

B. The Basic Concept of Net Present Value (NPV)

Net present value is a method of determining whether the cost of an investment is worth more or less than the value of the investment. NPV is calculated by subtracting the cost of the investment from the PV of the investment. The initial cost (IC) of an investment is the amount paid for the investment. NPV can be expressed as follows:

$$NPV = IC + PV$$

---

80. For a discussion of procedures for estimating the terminal FA, see infra part V.I.
81. For a discussion of the use of formulas in measuring an infinite series of free cash flows, see infra part V.J.
The algebraic formula for computing NPV can be written as follows:\(^{82}\)

\[
\text{NPV} = IC + \frac{1}{(1 + r)^n} (FA_n)
\]

IC is a negative number because it represents a cash outflow and

\[
\frac{1}{(1 + r)^n} (FA_n)
\]

is a positive number. Thus, if

\[
\frac{1}{(1 + r)^n} (FA_n)
\]

is greater than IC, the NPV of the investment is positive. These concepts are illustrated as follows:

Example (2). Assume that in Example (1) AC is able to purchase the stock of TC for $800,000 even though the value of TC to AC is $870,000. In that case the investment has a positive NPV of $70,000 computed as follows:

\[
\begin{align*}
\text{NPV} &= IC + \frac{1}{(1 + r)^n} (FA_n) \\
\text{NPV} &= (-$800,000) + \frac{1}{1 + .15} ($1 million) \\
\text{NPV} &= (-$800,000) + .870 ($1 million) \\
\text{NPV} &= (-$800,000) + $870,000 \\
\text{NPV} &= $70,000
\end{align*}
\]

Thus the value of AC should increase by $70,000 if it makes the acquisition.

C. The Basic Concept of Internal Rate of Return (IRR)

The internal rate of return is the rate of return that discounts expected future amounts (FAs) to an amount equal to the initial cost of an investment. Therefore, the IRR is the discount rate that produces a zero NPV.\(^{83}\) The IRR is the same as the con-

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82. BREALEY & MYERS, supra note 6, at 13.
83. Id. at 80.
cept of yield to maturity, which is used to determine the interest rate that can be expected from an investment in bonds.

Although there are potential problems with the use of IRR, generally if the IRR of an investment exceeds the cost of capital ($r$) of the investment, it makes good economic sense to make the investment. This is similar to saying that if the NPV is positive it makes economic sense to make the investment. The reason for the similarity is the obvious point that if the IRR of the investment exceeds the cost of capital of the investment, then the NPV has to be positive. The greater the NPV the more the IRR exceeds the cost of capital ($r$) of an investment.

In the case of an investment that is to be repaid in one year, the IRR can be determined from the following formula:

$$ IRR = \frac{\text{Expected Profit}}{\text{Cost of Investment}} $$

The computation is illustrated as follows:

**Example (3).** In Example (2) AC’s cost of the investment in the stock of TC is $800,000. Since the expected FA is $1 million, the expected profit is $200,000. Thus, the IRR is 25% (i.e., $200,000 ÷ $800,000). The IRR exceeds the cost of capital ($r$), which is 15%.

The IRR can also be computed using a computer or business calculator by following a three-step process: (1) enter the initial cost of the investment (IC); (2) enter each expected future payment (FA) for the period during which such payment is expected ($n$); (3) solve for the rate of interest that makes the present value of the expected future payments equal to the initial cost.

**D. The Relationship Between NPV and IRR**

In general, there is an equivalence between the NPV rule and the IRR rule in investment decision making. Under the NPV rule, companies will choose investments with a positive NPV. Under the IRR rule, companies will choose investments with an IRR that exceeds the cost of capital.

Notwithstanding the apparent equivalence between the NPV and IRR rules, it is commonly understood that the NPV method is superior to the IRR method and to other methods such as payback and average return on book value. The two rules reach the same result for conventional investments with a cash outflow in the first period followed by cash inflows in subsequent periods, such as with purchase of a bond. However, there are pitfalls associated with use of the IRR rule.

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84. *Id.* at 48.
85. *Id.* at 81-88.
86. *Brealey & Myers, supra* note 6, at 14.
87. *Id.* at 75; *see also Shapiro, supra* note 6, at 185-90.
88. These pitfalls are not explored here. They are discussed in *Brealey & Myers, supra* note 6, at 82-87; *see also Shapiro, supra* note 6, at 185-90.
E. Reasons for Use of Net Present Value in Making Investment Decisions

1. In General

Shareholder wealth is enhanced if the firm invests in projects with positive NPVs. Such projects add value to the firm because the expected returns exceed the firm's cost of capital. Since the firm can always distribute the cash to its shareholders and let them invest in the capital markets, the cost of capital is the opportunity cost of investing in a particular project, rather than in the capital markets. The cost of capital is the return shareholders could have earned had they invested the funds on their own in similar projects with comparable risk. Therefore, the discounting of a project's cash flows by the cost of capital measures how much investors would be prepared to pay for the project. This opportunity-cost concept is sensible when comparing assets of equivalent risk.

2. NPV and Economic Rents

Brealey and Myers explain that if a project has a positive NPV then in a classical economic sense the project produces economic rents. Economic rents are profits in excess of the firm's opportunity cost of capital. These rents may be either persistent, indicating that the firm has monopoly or market power, or temporary (i.e., quasi rents), indicating that the firm is not in long-run equilibrium. A positive NPV is nothing more than the present value of the economic rents.

Professor Shapiro makes a similar point:

The message from this analysis is clear: the run-of-the-mill firm operating in a highly competitive, commodity-type industry is doomed from the start in its search for positive net present value projects. Only firms that can bring to bear on new projects competitive advantages that are difficult to replicate have any assurance of earning excess returns in the long run.

Therefore, as a practical matter whenever there is a positive NPV the firm should have some identifiable advantage over other firms in the market. As Professor Shapiro says, "successful investments (those with positive NPVs) share a common characteristic: They are investments that create, preserve, enhance, and/or capitalize on competitive advantages that serve as barriers to entry."

Thus, acquiring corporations should be careful to avoid the tendency to be over optimistic in the preparation of cash flow estimates because such optimism can lead to
an overstated NPV. This type of over optimism is apparently one of the principal reasons for unsuccessful acquisitions. Of course, an overstated NPV may also result from an underestimate of the cost of capital, but for reasons that will become clear later, the cost of capital is generally based on standard financial processes and, therefore, is less susceptible to errors in calculation.

This optimism in the preparation of cash flow projections can lead to the "winner's curse,"\footnote{WESTON ET AL., MERGERS, supra note 2, at 254.} which can occur in an auction. Even though the average bidder in an auction may accurately assess the value of the thing being sold (e.g., the target), some bidders may underestimate the target's value and others may overestimate. The winner may be one who overestimates. Professor Roll argues that the winner's curse in acquisitions may result from hubris,\footnote{Richard Roll, The Hubris Hypothesis in Corporate Takeovers, J. Bus., Apr. 1986, at 197.} which is overconfidence by the managers of an acquiring corporation that their valuation of a target is correct and that the lower market valuation does not fully reflect the economic value of the target. Hubris may manifest itself in an overestimate of free cash flows.\footnote{WESTON ET AL., MERGERS, supra note 2, at 254-55.}

\section*{F. Capital Rationing and NPV}

In many cases the capital available for acquisitions or other investments is limited. And, even if capital is not limited, management capacity may restrict the amount of a firm's new investments.\footnote{Id. at 56.} A firm may have multiple acquisition or investment opportunities that have positive NPVs. In such cases the firm must decide which of the various opportunities to pursue. Thus, the firm is faced with choosing among several investment opportunities all of which produce a positive NPV. If capital and management resources were unlimited, the firm would invest in each project.\footnote{Id. at 56.}

One way of solving this capital rationing problem is to choose the project or projects with the highest profitability index.\footnote{BREALEY & MYERS, supra note 6, at 113-14; see also SHAPIRO, supra note 6, at 192-93.} The profitability index is the ratio of the present value of the project to the initial cost:

\[
\text{Profitability Index} = \frac{\text{Present Value of Project}}{\text{Initial Cost of Project}}
\]

If the profitability index for a project is one, the project's NPV is zero because the present value of the project equals the initial cost. The more the profitability index exceeds one, the greater the profitability of the project. For example, a project with a profitability index of 1.41 returns $1.41 in present value for each dollar of investment.\footnote{SHAPIRO, supra note 6, at 190.} Thus, selection of projects with the highest profitability index produces the greatest present value return on investment. There are several more elaborate capital rationing techniques.\footnote{BREALEY & MYERS, supra note 6, at 114-17.}
IV. PRESENT VALUE OF MULTIYEAR INCOME FLOWS: THE DISCOUNTED CASH FLOW AND NET PRESENT VALUE FORMULAS

A. Introduction

The formula for computing the present value of a cash flow is as follows:

\[ PV = (\text{Discount Factor}_n)(FA_n) \]

\[ PV = \frac{1}{(1 + r)^n}(FA_n) \]

This assumes that the required rate of return \( r \) is the same for each period \( n \); that is, the required rate for discounting an FA to be received in year two is the same as the required rate that is used for discounting an FA to be received in year ten. As will be seen in the discussion of CAPM, the \( r \) is generally assumed to be the same for all periods. However, because of the term structure of interest rates, which shows the relationship between short-term rates and long-term rates, it may be appropriate to use a different discount rate for each period.\(^{106}\)

Although \( r \) is assumed to be the same each period, the discount factor (i.e., \( \frac{1}{(1 + r)^n} \)) is different for each period. The greater the time until a particular payment date, the smaller the discount factor.

Assuming that future payments (FAs) are to be received at the end of years one, two, and three, the present value or discounted cash flow (DCF) formula can be written as follows:

\[ PV = \frac{FA_1}{(1 + r)} + \frac{FA_2}{(1 + r)^2} + \frac{FA_3}{(1 + r)^3} \]

The shorthand expression for the DCF formula is:\(^{107}\)

\[ PV = \sum \frac{FA_n}{(1 + r)^n} \]

Interest is automatically compounded in the DCF formula.\(^{108}\) The assumption in this model (which is often the assumption in capital budgeting)\(^{109}\) is that cash flows (FAs)

\(^{106}\) COPELAND & WESTON, supra note 6, at 70-71. An illustration shows that the NPV of two different projects can differ depending on whether the cash flows are discounted at a single rate or the rates implied in the term structure. Id. at 71.

\(^{107}\) BREALEY & MYERS, supra note 6, at 30.

\(^{108}\) Id. at 37.

\(^{109}\) Id. at 41.
are paid at the end of the year. This need not be the case, however. For example, interest on U.S. bonds is generally paid semiannually.\footnote{110} If FAs are paid more frequently than annually, the annual yield \((r)\) must be adjusted to reflect such payments. For example, if the yield to maturity on a bond is 10\% annually and the bond makes semiannual payments, the semiannual yield \((r)\) is 5\%.

The net present value is determined by merely adding to the DCF formula the initial cost of the investment\footnote{111} as follows:

\[
\text{NPV} = IC + PV = IC + \sum \frac{FA_n}{(1 + r)^n}
\]

In essence, this involves subtracting from the present value of the cash flows the initial cost of the investment.

**B. The Five Basic Steps in Using DCF and NPV**

There are five basic steps in using the DCF and NPV concepts.\footnote{112} First, the analyst must estimate the amount and timing of the income stream,\footnote{113} or the negative income stream\footnote{114} an investment will generate for each year of the investment’s life.\footnote{115} This requires an estimate of the terminal value of the investment. The terminal value of the investment is the amount to be realized when the investment is finally disposed of or liquidated. The analyst simplifies the calculation of the present value of what is otherwise an infinite series of cash flows by using a terminal value.

An investment may have a negative cash flow for a particular year. A negative cash flow occurs when the required investment of cash for that year exceeds the net cash flow generated from operations for that year. Second, the analyst must determine the appropriate discount rate for the particular investment. Although there are many techniques for determining the discount rate, three leading techniques are discussed here: (1) the capital asset pricing model (CAPM),\footnote{116} (2) the arbitrage pricing theory (APT),\footnote{117} and (3) the weighted average cost of capital (WACC).\footnote{118} Section IX demonstrates that the cost of equity in the WACC model may be determined through the use of CAPM or APT.

Third, the analyst must discount the free and negative cash flows for each period and the terminal value to present value using the DCF formula. The discounting of a negative cash flow results in a negative present value for that particular flow. Fourth,

\footnote{110}{Id. at 38.}
\footnote{111}{IC is a negative number because it is a present cash outflow.}
\footnote{112}{These five steps are based on the four-step process outlined in BREALEY & MYERS, supra note 6, at 73.}
\footnote{113}{Income stream accounts for free cash flow, not accounting profits.}
\footnote{114}{Negative income stream means negative cash flow.}
\footnote{115}{See infra part V (exploring the estimation process).}
\footnote{116}{See infra part VI (discussing the use of CAPM).}
\footnote{117}{See infra part VII (discussing the use of APT).}
\footnote{118}{See infra part IX (discussing the use of WACC).}
the analyst must determine the present value of the investment by adding the positive present values of the free cash flows and the terminal value and subtracting the aggregate of the negative present values of the negative cash flows. This amount is represented algebraically as follows:

$$\sum \frac{FA_n}{(1 + r)^n}$$

$FA_n$ represents both the free or negative cash flow for each period and the terminal value for the last period. The discount rate is $r$.

Finally, to compute the NPV of the investment, the analyst must subtract the initial cost (IC) of the investment from the present value of the investment. The NPV formula is as follows:

$$NPV = IC + \sum \frac{FA_n}{(1 + r)^n}$$

$$NPV = IC + \frac{FA_1}{(1 + r)^1} + \frac{FA_2}{(1 + r)^2} + \frac{FA_3}{(1 + r)^3} + ... + \text{Terminal Value}$$

If the result is positive, then the investment has a positive NPV and should be undertaken unless there are other projects with positive NPVs and the firm's capital rationing technique, such as the profitability index, indicates that the firm should undertake one or more of such other projects instead.¹¹⁹

If the target has multiple lines of business, the acquiror should follow this five-step process in valuing each line, with a separate discount rate determined for each line of business.

This process can be illustrated as follows:

**Example (4).** Acquiring corporation (AC) is considering the acquisition of all of the assets of target corporation (TC). TC is asking $1.5 million for all the assets, and this is a firm, non-negotiable price. TC has no outstanding debt. AC expects to dispose of the acquired assets three years after the acquisition. AC uses a five-step process in determining whether to make this investment:

First, AC estimates that TC's assets will generate $100,000 of free cash flow at the end of the first year of operation and $400,000 at the end of the second year. At the end of the third year AC expects to sell TC for $2 million. AC does not foresee any free cash flow from the operation of TC during the third year.

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¹¹⁹. See supra part III.F.
Second, using CAPM, APT, or WACC, AC determines that the appropriate discount rate for an investment in TC’s assets is 15%.

Third, AC determines the present value of the $100,000 and $400,000 free cash flows and of the $2 million terminal value:

\[
PV = \sum \frac{1}{(1 + r)^n} (FA_n)
\]

\[
PV = \frac{1}{(1 + .15)} ($100,000) + \frac{1}{(1 + .15)^2} ($400,000) + \frac{1}{(1 + .15)^3} ($2,000,000)
\]

\[
PV = .870($100,000) + .756($400,000) + .658($2 million)
\]

\[
PV = $87,000 + $302,400 + $1,316,000
\]

Fourth, AC adds the present values of the cash flows and the terminal value. This results in a present value of the investment of $1,705,400:

<table>
<thead>
<tr>
<th>Present Value of</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>87,000</td>
</tr>
<tr>
<td>$400,000</td>
<td>302,400</td>
</tr>
<tr>
<td>$2,000,000</td>
<td>1,316,000</td>
</tr>
<tr>
<td></td>
<td>$1,705,400</td>
</tr>
</tbody>
</table>

The same computation could be made on a business calculator by entering each of the free cash flow amounts and the terminal value for the applicable period and then discounting these figures to present value at the 15% discount rate.

Finally, in order to compute the net present value of the investment, AC subtracts from the present value of the investment the initial cost of the investment, which is the $1.5 million asking price for the shares. Thus, the investment has an NPV of $205,400 and should be undertaken, unless there are other potential projects with positive NPVs and the firm’s capital rationing model indicates that one or more of those other projects should be pursued instead of this one.

C. Use of DCF and Other Methodologies in Time’s Acquisition of Warner

The use of the DCF methodology in Time’s acquisition of Warner is illustrated in the Wasserstein/Shearson presentation to the Time board and in Lazard’s presentation to the Warner board. Wasserstein/Shearson estimated the free cash flow

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120. See supra part I.A (providing chart and discount factors).
121. See infra Appendix C.
122. See infra Appendix D.
amounts for Warner's Film Entertainment segment. The present value of the Film Entertainment segment was computed using three different discount rates and three different estimates of the terminal value. However, the appendix does not indicate how the range of discount rates was determined. The different terminal values are apparently determined by multiplying the projected free cash flow amount for the terminal year by one of four multipliers. A similar DCF process is followed for each of Warner's other business segments.

Wasserstein/Shearson also utilized the comparable transaction technique and the comparable company technique in valuing each of Warner's business segments. The Wasserstein/Shearson summary of the pre-tax segment valuations of each of Warner's business segments utilizing DCF, comparable acquisitions, and comparable companies is included in Appendix C. This is a valuation of the operating assets of each segment without taking account of associated debt. Appendix C also shows how the range of asset values for the various business segments are adjusted to produce an estimate of the equity value of Warner. On a per share basis, this estimate ranges from a low of $68.40 to a high of $77.12. In computing equity value, cash and investments, which are not included in the DCF analysis, are added to the asset value, and debt is subtracted. The reason for first valuing the assets and then valuing the equity by, among other things, subtracting the outstanding debt is discussed below.

Lazard prepared a DCF analysis of potential prices of Warner's stock for 1991 and 1992, discounted back to June 30, 1989. Three different equity discount rates are utilized in the discounting. In determining the expected terminal value, three different price/earning multiples are utilized for determining the expected prices of Warner stock in 1991 and 1992. Lazard presented a consolidated unleveraged DCF analysis of Warner. In this analysis the expected free cash flows for years 1989 to 1994 are set forth, and these amounts, together with the expected terminal value, are discounted to present value as of June 30, 1989. Five different discount rates are utilized. A range of terminal values is obtained by utilizing three different multiples of expected free cash flow for the terminal year (1994).

Lazard determined a range of values for Warner's Film Entertainment segment based upon a public market range, a private market range, and a discounted cash flow

123. See infra Appendix C, page C-2; see also infra part V (discussing the estimation process).
124. See infra Appendix C, page C-3.
125. For a discussion of this method of determining terminal value, see infra part V.I.7.
126. See, e.g., Appendix C, page C-4.
127. See, e.g., Appendix C, page C-5.
128. See infra Appendix C, page C-6.
129. See infra part V.E (discussing the reasons for separating the investment decision from the financing decision).
130. See infra Appendix C, page C-7.
131. See infra part VI.F (discussing the difference between asset betas and equity betas).
133. For a discussion of the price/earning method of determining terminal value, see part V.I.4.
134. See infra Appendix D, pages D-2 to D-3.
135. See infra part V.I.7 (discussing the estimation of terminal value in the Time-Warner acquisition).
range.\textsuperscript{136} Similar valuation ranges were provided for Warner's other segments. Both the Wasserstein/Shearson and the Lazard valuation methodologies are discussed in the excerpts from the Time/Warner Information Statement and Time Prospectus.\textsuperscript{137}

V. ESTIMATING FREE (OR NEGATIVE) CASH FLOWS AND TERMINAL VALUE

A. Introduction

The following are two key elements in the DCF and NPV techniques: (1) estimation of the cash flows and terminal value, and (2) determination of the appropriate discount rate. This section provides some guidelines for estimating the periodic free or negative cash flows from a project and the terminal value of a project. Unless an infinite series of cash flows is projected,\textsuperscript{138} both periodic cash flows and a terminal value are needed for the DCF model to function. Sections V, VI, and VII deal with the second key element of the DCF and NPV models: determination of the discount rate.

B. General Principles

Four general rules govern the preparation of a cash flow statement. First, the only relevant factor is cash flow; thus, accounting earnings are irrelevant. Second, cash flow is estimated on an incremental basis, meaning that only the cash flows resulting from the investment are taken into account. Third, the investment decision is separated from the financing decision. Fourth, inflation must be treated consistently.\textsuperscript{139}

C. Estimating Cash Flows\textsuperscript{140}

Cash flow is the difference between "dollars-in" and "dollars-out."\textsuperscript{141} Cash flow is different from the accounting earnings or profits of a firm.\textsuperscript{142} In computing profits, accountants may defer reporting certain cash receipts to future periods or report in an earlier period certain expected future cash receipts. Also, accountants divide cash outflows between deductible expenses and capital expenditures, which are recovered through depreciation deductions. Thus, accounting profits are not the same as cash flow.

After-tax cash flow is determined by deducting taxes in the year actually paid, not when the liability accrues.\textsuperscript{143} The taxes are computed as if the firm had no debt and, therefore, no deductible interest. Taxes are determined on the basis of the projected

\textsuperscript{136} See infra Appendix D, pages D-5 to D-6.

\textsuperscript{137} See infra Appendix A.

\textsuperscript{138} See infra part V.J.

\textsuperscript{139} BREALEY & MYERS, supra note 6, at 96.

\textsuperscript{140} For an elaborate discussion of the details of estimating free cash flows, see COPELAND ET AL., VALUATION, supra note 6, at 109-69; CORNELL, CORPORATE VALUATION, supra note 6, at 108-43; see also Keinwort Benson LTD. v. Silgan Corp., 1995 WL 376911, at *5-6 (Del. Ch. 1995) (discussing estimation of free cash flows); SHAPIRO, supra note 6, at 204-30; Gale & Branch, Cash Flow Analysis: More Important Than Ever, HARV. BUS. REV. 131 (July-August 1981).

\textsuperscript{141} BREALEY & MYERS, supra note 6, at 96.

\textsuperscript{142} Id.

\textsuperscript{143} Id.
actual tax liability\textsuperscript{144} (not the book tax liability) for an all-equity financed transaction.\textsuperscript{145} Thus, cash flow is computed without taking account of the manner in which the project is financed. The result of this computation is called the unlevered cash flow. This means that the cash flow does not take into account any tax savings from interest deductions.\textsuperscript{146}

One of two methods is used to account for interest deductions. The first method is the adjusted net present value method (APV).\textsuperscript{147} Under the APV method, projected tax savings from interest deductions are separately determined and then are discounted back to present value. The present value of tax savings is then added to the NPV of the project, which yields APV. The second method uses the weighted average cost of capital (WACC) to determine the discount rate.\textsuperscript{148} Any tax savings from the use of debt is automatically accounted for by the use of a lower discount rate. Correctly used, these two methods should produce the same result.

Noncash deductions that are allowed in computing earnings, such as depreciation and amortization deductions, are added back in computing cash flow. Cash expenditures that are not deductible in computing earnings, such as capital expenditures, are deducted in computing cash flow. Thus, computing cash flow is merely a process of subtracting dollars going out from dollars coming in.

Also, the working capital needs of the firm should be recognized as a cash expenditure in estimating cash flows.\textsuperscript{149} Working capital is the excess of cash investments, accounts receivable, and inventory over accounts payable. Working capital should grow with growth of the firm.

The only exception to the "dollars-in" rule is that the proceeds of debt and equity financing are not included as cash in; the only exception to the "dollars-out" rule is that interest payments to debt holders and dividends to shareholders are not deducted. The reason for these two exceptions is that under the DCF model,\textsuperscript{150} the financing decision is separated from the investment decision.

The above principles are set forth in the following equation:

\[
\text{Free Cash Flow (FCF)} = (\text{Revenue}) - (\text{Non-Interest Expense, including Depreciation}) - (\text{Taxes}) + (\text{Depreciation}) - (\text{Working Capital}) - (\text{Capital Expenditures}).
\]

A positive free cash flow (FCF) indicates the amount of funds available to pay the providers of capital.\textsuperscript{151} A negative FCF indicates the amount of funds that must be made available to the firm by the providers of capital.\textsuperscript{152} In a normal investment, the capital providers fund the initial period investment, which is a negative cash flow. The capital providers receive the subsequent period free cash flows as compensation for

\textsuperscript{144} Id.
\textsuperscript{145} Id. at 106-07.
\textsuperscript{146} The unlevered cash flow is illustrated infra at Appendix C, page C-2.
\textsuperscript{147} See infra part VIII (discussing the adjusted net present value technique).
\textsuperscript{148} See infra part IX (discussing the use of WACC).
\textsuperscript{149} BREALEY & MYERS, supra note 6, at 97.
\textsuperscript{150} See infra part VIII (discussing the interaction of the investment decision and the financing decision).
\textsuperscript{151} The providers of capital include both shareholders and creditors.
\textsuperscript{152} COPELAND ET AL., VALUATION, supra note 6, at 111.
providing the initial investment. If the present value of the subsequent period free cash flows exceeds the amount of the initial negative free cash flow, the project has a positive net present value. In that case, the capital providers can expect to receive their investment back plus an adequate return.

D. Estimating Cash Flows on an Incremental Basis

In projecting cash flows only additional flows that arise because of the project are considered. For example, accounting conventions may allocate part of the overhead cost of an acquiring corporation, such as supervisory salaries, rent, heat, and lights, to a newly acquired target corporation. However, in projecting the cash flows expected to be generated by the target, only the extra overhead cost that can be expected to result from the acquisition of the target should be included. Thus, in computing free cash flows in the acquisition context, only those additional cash flows directly attributable to the acquisitions are taken into account.

E. Separating the Investment Decision from the Financing Decision

The investment decision is separate from the financing decision in applying the DCF model. Therefore, debt proceeds used to make the investment are ignored and interest and principal payments on debt are not treated as cash flows. Thus, the project is treated as if it were completely financed with equity, with all cash outflows coming from shareholders and all cash inflows going to them. The financing decision is made only after a computation of NPV.

Although the impact of debt acquisition financing is ignored in applying the DCF methodology, the amount of debt that can be used to finance the transaction is in a real way constrained by the free cash flow available for the payment of interest and principal on the debt. This takes into account, of course, the deductibility of interest, which reduces the tax liability.

The analysis of the debt repayment ability after an acquisition is often done by comparing (1) the operating cash flow, which can be estimated using earnings before depreciation, interest, and taxes (EBDIT), with (2) the interest and debt repayment costs. Other analyses can be undertaken, as well, including a comparison of the balance sheets, operating costs, and earnings per share of the acquiring corporation after the acquisition under various financing proposals. Other analyses include the assumption of an acquisition with 50% cash and 50% stock.

The cost and benefits associated with the financing decision, such as issuance cost and the tax savings from interest deductions, are separately determined and then dis-

153. BREALEY & MYERS, supra note 6, at 96-98.
154. Id. at 98.
155. Id. at 103.
156. Id.
157. See infra Appendix C, at C-8 (illustrating this type of financing analysis in the context of Time's acquisition of Warner; containing a portion of Wasserstein/Shearson's analysis of the financial effects of an all cash acquisition of Warner).
counted back to present value.\textsuperscript{158} This process is not used, however, if the discount rate is determined by using WACC, which takes into account the financing decision.

\textbf{F. Treating Inflation Consistently}

The projected cash flows should reflect the projected effects of inflation because the discount rate generally reflects such effects. The discount rate is stated in nominal terms rather than real terms,\textsuperscript{159} and it would be inconsistent to discount cash flows stated in real terms at the nominal rate. Thus, it is important to state both the cash flows and the discount rate on a nominal basis.\textsuperscript{160} This means that an estimate of inflation must be taken into account in making the cash flow projections.

\textbf{G. Illustration of Estimation of Free (or Negative) Cash Flows}

In estimating cash flows, the starting point is to estimate the target's inflation-adjusted sales for each year, running from the acquisition date to the termination date. This sales forecast often is the most important part of a cash flow forecast.\textsuperscript{161} All other elements in the cash flow projection are driven by projected sales. The inflation-adjusted cost of goods sold and other operating expenses are then subtracted from the sales to arrive at operating profit. This leaves earnings before interest and taxes (EBIT). Next, taxes are subtracted from operating profit to reach an estimate of unleveraged net cash flow from operations, that is, cash flow without taking into account any debt. Any noncash deductions included in operating expenses, such as depreciation, amortization of goodwill, and deferred taxes, are added to this amount.

Finally, any needed inflation adjusted capital investments and inflation-adjusted increases in working capital are subtracted. The resulting figure is unleveraged free cash flow. Wasserstein/Shearson followed this process in their presentation to Time's board.\textsuperscript{162} The process is illustrated in the following example:

\textit{Example (5).} Acquiring corporation (AC) is considering the acquisition of target corporation (TC). AC has prepared the following inflation-adjusted estimates of TC's free cash flow for the five-year period after the date of the acquisition. AC plans to dispose of TC at the end of the five-year period.
<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Sales [Minus]</td>
<td>$1,600,000</td>
<td>$1,700,000</td>
<td>$1,800,000</td>
<td>$1,900,000</td>
<td>$2,100,000</td>
</tr>
<tr>
<td>(2) Cost of Goods Sold [and]</td>
<td>350,000</td>
<td>475,000</td>
<td>500,000</td>
<td>575,000</td>
<td>650,000</td>
</tr>
<tr>
<td>(3) Other Operating Expenses (Including Depreciation) [Equals]</td>
<td>150,000</td>
<td>125,000</td>
<td>100,000</td>
<td>75,000</td>
<td>50,000</td>
</tr>
<tr>
<td>(4) Operating Profit [Minus]</td>
<td>1,100,000</td>
<td>1,100,000</td>
<td>1,200,000</td>
<td>1,250,000</td>
<td>1,400,000</td>
</tr>
<tr>
<td>(5) Taxes [Equals]</td>
<td>300,000</td>
<td>300,000</td>
<td>350,000</td>
<td>350,000</td>
<td>350,000</td>
</tr>
<tr>
<td>(6) Unleveraged Net Cash Flow [Plus]</td>
<td>800,000</td>
<td>800,000</td>
<td>850,000</td>
<td>900,000</td>
<td>1,050,000</td>
</tr>
<tr>
<td>(7) Depreciation [and]</td>
<td>100,000</td>
<td>75,000</td>
<td>50,000</td>
<td>25,000</td>
<td>--</td>
</tr>
<tr>
<td>(8) Amortization of Goodwill [and]</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>(9) Deferred Taxes [and]</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
</tr>
<tr>
<td>(10) Any other Noncash Expenses [Minus]</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
</tr>
<tr>
<td>(11) Capital Expenditures [and]</td>
<td>300,000</td>
<td>165,000</td>
<td>90,000</td>
<td>??</td>
<td>??</td>
</tr>
<tr>
<td>(12) Increases in Working Capital [and]</td>
<td>10,000</td>
<td>20,000</td>
<td>20,000</td>
<td>35,000</td>
<td>60,000</td>
</tr>
<tr>
<td>(13) Any Other Non-Deductible Cash Expenditures [Equals]</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
</tr>
<tr>
<td>(14) Unleveraged Free Cash Flow</td>
<td>$ 600,000</td>
<td>$ 700,000</td>
<td>$ 800,000</td>
<td>$ 900,000</td>
<td>$ 1,000,000</td>
</tr>
</tbody>
</table>
H. Use of a Formula in Estimating Free Cash Flows

If specific projections of the elements of a target’s free cash flows are not available, a projected cash flow statement may be prepared by using a formula that makes certain assumptions about (1) the expected growth in sales, (2) the operating profit margin, (3) the incremental fixed capital investment rate, and (4) the incremental working capital investment rate. This method may be used, for example, when the acquiring corporation does not have access to the target’s internal cash flow projections. Professor Rappaport states the formula as follows:

\[
\text{Cash Flow} = (\text{Cash In}) - (\text{Cash Out}) \\
= [(\text{Sales in Prior Year})(1 + \text{Sales Growth Rate}) \\
(\text{Operating Profit Margin})(1 - \text{Cash Income Tax Rate})] \\
- [(\text{Increase in Sales})(\text{Incremental Fixed Capital Investment Rate}) \\
+ (\text{Increase in Sales})(\text{Incremental Working Capital Investment Rate})]
\]

The “Cash In” is determined by multiplying sales in the prior year by \((1 + \text{Sales Growth Rate})\), producing the expected sales for the next year. This amount multiplied by the Operating Profit Margin produces the expected operating profit for the year. The Operating Profit Margin is the ratio of pre-interest, pre-tax operating profit to sales. By focusing on pre-interest operating profit, the formula properly does not take interest expense into account in computing free cash flow.

The expected operating profit includes allowances for depreciation, but depreciation is eliminated from the formula as discussed below. The expected operating profit multiplied by \((1 - \text{Cash Income Tax Rate})\) gives the expected after-tax earnings for the year, or “Cash In.” The term \((1 - \text{Cash Income Tax Rate})\) gives the after-tax rate. For example, if the Cash Income Tax Rate is 34%, the after-tax rate is 66%. The actual tax rate rather than the book rate is used.

The sum of the two Cash Out amounts are deducted from the expected after-tax earning (Cash In) to come up with free cash flow. The first deductible amount is the Incremental Fixed Capital Investment, which is estimated by multiplying the expected Increase in Sales for the year by the historical Incremental Fixed Capital Investment Rate for the business. This represents the historical relationship between increase in sales and increase in capital investment. Incremental Fixed Capital Investment takes into account capital expenditures minus depreciation expense, thereby eliminating deprecia-

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163. RAPPAPORT, supra note 6, at 51-55.
164. Id.
165. For example, if sales are $10 million and all expenses except interest and taxes are $8 million, the operating profit is $2 million and the Operating Profit Margin is 20%.
166. Expected Operating Profit = (Sales in Prior Year)(1 + Sales Growth Rate)(Operating Profit Margin).
tion from the formula. The Incremental Fixed Capital Investment Rate is computed by dividing Incremental Fixed Capital Investment by Incremental Sales:

\[
\text{Incremental Fixed Capital Investment Rate} = \frac{(\text{Capital Expenditures}) - (\text{Depreciation Expense})}{\text{Incremental Sales}}
\]

The second deductible amount is the Incremental Working Capital Investment, which is estimated by multiplying the expected Increase in Sales for the year by the historical Incremental Working Capital Investment Rate. This represents the historical relationship between the Increase in Sales and the increase in working capital. The Incremental Working Capital Investment, which "represents the net investment in accounts receivable, inventory, accounts payable and accruals that are required to support sales growth," is expressed as a percentage of sales:

\[
\text{Incremental Working Capital Rate} = \frac{\text{Incremental Working Capital Investment}}{\text{Incremental Sales}}
\]

Adjustments to this formula may be required for adding back amortization and other noncash deductions.

Example (6). The above formula can be illustrated as follows. Assume the following basic information concerning the target:

<table>
<thead>
<tr>
<th>itty in Prior Year:</th>
<th>10,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Growth Rate:</td>
<td>10%</td>
</tr>
<tr>
<td>Operating Profit Margin:</td>
<td>10%</td>
</tr>
<tr>
<td>Cash Income Tax Rate:</td>
<td>40%</td>
</tr>
<tr>
<td>Incremental Fixed Capital Investment Rate:</td>
<td>10%</td>
</tr>
<tr>
<td>Incremental Working Capital Investment Rate:</td>
<td>5%</td>
</tr>
</tbody>
</table>

Historical data are used in estimating the target's Sales Growth Rate, Operating Profit Margin, Incremental Fixed Capital Investment Rate, and Incremental Working Capital Investment Rate.

In computing expected Cash In for next year, the Sales in Prior Year of $10 million are multiplied by \((1 + \text{Sales Growth Rate})\), or 110%, to come up with the expected sales in the next year, $11 million. This number is

167. RAPPAPORT, supra note 6, at 54.
multiplied by the Operating Profit Margin of 10% to come up with the expected operating profit for next year of $1.1 million. This amount is multiplied by (1 - Cash Income Tax Rate of 40%), which produces an expected Cash In for next year of $660,000.

In computing expected Cash Out for next year, the expected Increase in Sales for next year of $1 million is multiplied by the Incremental Fixed Capital Investment Rate of 10% to get the expected increase in investment of $100,000. This means that the target can be expected to make capital expenditures of $100,000 in excess of depreciation for the next year.

Next, in computing Cash Out, the $1 million expected Increase in Sales for next year is multiplied by the Incremental Working Capital Investment Rate of 5% to get an expected $50,000 increase in working capital for next year. This increase in working capital is needed to support the $1 million increase in sales. The expected increase in capital investment for next year of $100,000 plus the expected increase in working capital for next year of $50,000 are added to equal an expected Cash Out for next year of $150,000.

Finally, the expected Cash Out of $150,000 is deducted from the expected Cash In of $660,000 to equal an expected operating cash flow for next year of $510,000. Amortization or other noncash deductions would be added to this amount.

Professor Rappaport used this model with some variation in his valuation study in the Technicolor case. The court describes his approach as follows:

Again a discounted cash flow analysis was used [by Professor Rappaport's firm Alcar.] North Hollywood’s net operating profit during the forecast period was estimated through a process informed by history and judgment. First a base (1983) year forecast was established, and then assumptions about growth during the forecast period were made.

For his base forecast, Professor Rappaport adopted management’s projections for calendar year 1983 sales at North Hollywood. Alcar then assumed that the company would achieve a 19% operating profit margin during 1983. The 19% figure was based on historic margins at North Hollywood. Alcar normalized the historic data in a responsible way to eliminate the effect that abnormal silver reclamation profits had had upon North Hollywood’s profit margin during fiscal years 1980 and 1981. From the sales and profit margin figures, Alcar derived the costs and net operating profit projections for North Hollywood. For years 1984-1987, Alcar adjusted its sales estimate to account for the loss of United Artists as a client, and otherwise assumed flat (zero) growth. Sales were assumed to be constant by Alcar for reasons that on balance I find reasonable and operating profits were estimated to remain at 19% throughout the forecast period.168

Wasserstein/Shearson also used a similar process in estimating Warner’s Cash In and Cash Out for 1991 through 1998. In the discussion of the Wasserstein/Shearson valuation in the Joint Time and Warner Information Statement and Time Prospectus, it is explained that the cash flow projections for Warner were based on “certain estimated financial information for 1989 and 1990 provided by [Wasserstein/Shearson] by [Warner] and on extrapolations of such projections through 1998 performed by [Wasserstein/Shearson].”

Weston et al., in Mergers, Acquisitions, and Corporate Control, criticize the measurement of profits under the Rappaport formula and demonstrate how the formula can be restated in a more precise mathematical model. As a practical matter, if the profit margin adequately expresses profits for a firm, the Rappaport model is appropriate and understandable by those not heavily schooled in mathematics. For those who are comfortable with mathematics, the Weston et al. model may be more appropriate.

I. Estimating Terminal Value

1. Introduction

The DCF model does not work unless there is either an estimate of the final sales or liquidation value of the target or an infinite series of periodic cash flows. In most cases involving a target, the parties estimate the cash available from the final disposition of the target. This was the method used by both Lazard and Wasserstein/Shearson in their valuation of Warner. The terminal date is often chosen arbitrarily, and terminal (or residual or horizon) value is often the largest portion of the firm’s value. Professor Rappaport points out that “for most businesses only a small proportion of value can be reasonably attributed to its estimated cash flow for the next five or ten years.”

Brealey & Myers discuss two models and two rules of thumb for determining terminal value: (1) the constant growth model, (2) the competitive equilibrium

170. See infra Appendix A, at A-4.
171. Id.
172. WESTON ET AL., MERGERS, supra note 2, at 159-61; see also infra part V.J (discussing the mathematical models in WESTON ET AL., MERGERS).
173. See infra part V.J (discussing formulas that can be used in the DCF model when there is an infinite series of cash flows).
174. See infra Appendix C, at C-3 (Wasserstein/Shearson); Appendix D, at D-4 (Lazard).
175. BREALEY & MYERS, supra note 6, at 64.
176. See infra Appendix C, at C-3 (showing that under all assumptions, the present value of the terminal value exceeds the present value of the free cash flows).
178. BREALEY & MYERS, supra note 6, at 64-67.
179. See infra part V.I.2.
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model,"(3) the price/earnings rule of thumb,\textsuperscript{181} and (4) the market/book rule of thumb.\textsuperscript{182}

Brealey and Myers state that "[a] rule of thumb, artfully employed, sometimes beats a complex discounted cash flow calculation hands down,"\textsuperscript{183} acknowledging the imperfections in the DCF process. After a discussion of the above techniques,\textsuperscript{184} this Article discusses the multiple of cash flow technique.\textsuperscript{185} Finally, this Article discusses the use of mathematical formulas for measuring an infinite series of free cash flows.\textsuperscript{186}

None of the methods for estimating terminal value involves an explicit estimate of terminal free cash flow. Thus, one may ask why the methods for estimating terminal value are not used in the current time period to determine the present value of the target. In the words of Professor Cornell, "Why not define the current time to be the terminal date and value the firm today using whatever procedure is employed to estimate the continuing value? Such a simplification would eliminate both the need to forecast cash flows and the need to calculate present values."\textsuperscript{187} Professor Cornell explains that "simplified procedures are applicable only when the firm has reached an 'equilibrium state' and is no longer evolving rapidly."\textsuperscript{188} Professor Cornell then adds the following important insight: "By postponing the date at which the simplified models are applied, the impact of errors produced by these models is mitigated by the discounting process."\textsuperscript{189}

As will be seen below, the different models for estimating terminal value can produce dramatically different results. On this point Professor Cornell states:

Given the strengths and weaknesses of each procedure, it is wise to use more than one model to calculate the continuing value. If the different models produce similar continuing values, confidence in the estimate rises. If the different models produce different values, further research is suggested to reconcile the discrepancies.\textsuperscript{190}

### 2. Estimating Terminal Value Through the Constant Growth Model

The constant growth formula first requires an estimate of the free cash flow for the first year in which the target is likely to settle into a long-run growth rate. This requires an estimate of the point at which the target can be expected to grow at a constant

\textsuperscript{180} See infra part V.1.3.
\textsuperscript{181} See infra part V.1.4.
\textsuperscript{182} See infra part V.1.5.
\textsuperscript{183} BREALEY & MYERS, supra note 6, at 66.
\textsuperscript{184} See infra parts V.1.2-6.
\textsuperscript{185} See infra part V.1.7. This technique was utilized by both Lazard and Wasserstein/Shearson in their valuation of Warner.
\textsuperscript{186} See infra part V.1.
\textsuperscript{187} CORNELL, CORPORATE VALUATION, supra note 6, at 144.
\textsuperscript{188} Id.
\textsuperscript{189} Id.
\textsuperscript{190} Id. at 168
rate. The second element in the formula is an estimate of the rate of long-term growth of free cash flow. The terminal value is then determined pursuant to the following formula:

\[
\frac{(FCF)}{(r) - (g)}
\]

For this formula to work, the discount rate must exceed the long-term growth rate. This formula is similar to the constant-dividend growth valuation model often used by analysts in valuing marketable securities. In that model, the numerator is the expected dividend, and the denominator is the cost of equity capital minus the dividend growth rate.

The constant growth model is illustrated as follows:

Example (7). Acquiring corporation estimates that five years after the acquisition of the target corporation (TC), TC’s inflation-adjusted free cash flow will settle into a long-term growth pattern at an expected growth rate of 5%. The estimated free cash flow for year five is $1 million. Thus, in year six, for example, free cash flow is expected to be $1,050,000 (i.e., $1 million x 1.05). The required discount rate is 15%. The estimated terminal value (TV) of TC at the end of year five, therefore, is $10 million computed as follows:

\[
\begin{align*}
\text{Free Cash Flow in Year 5} &= \frac{\$1,000,000}{0.15 - 0.05} \\
&= \frac{\$1,000,000}{0.10} \\
&= \$10,000,000
\end{align*}
\]

The denominator of .10 is the equivalent of a price/earning multiple of ten, that is $1 million/.10 is the same as 10 times $1 million. The .10 is sometimes referred to as the capitalization rate, the rate used in determining the

191. BREALEY & MYERS, supra note 6, at 64.
192. INVESTMENTS, supra note 6, at 474-77.
capital value of the income stream produced by an investment. The present value of this $10 million terminal value is computed as follows:  

\[
\frac{1}{(1 + r)^n} (TV_m)
\]

\[
\frac{1}{(1 + .15)^5} ($10,000,000)
\]

\[
.497 \times $10 \text{ million} = $4,970,000
\]

This process involves two discounting steps. First, the free cash flows for the terminal year are discounted at a rate equal to the cost of capital minus the growth rate to determine the terminal value. Second, the terminal value is discounted at the cost of capital to arrive at the present value of the terminal value.

This constant growth model for estimating terminal value was utilized by the plaintiff's valuation expert and accepted by the Delaware Chancery Court in Radiology Associates. The court accepted this method without discussing the advisability of using other methods for determining terminal value.

3. Estimating Terminal Value Through the Competitive Equilibrium Model

The competitive equilibrium model requires a determination of when the "industry is likely to settle into competitive equilibrium," that is, when competitors catch up and everyone is on an equal footing. At that point the net present value of new growth opportunities is zero. The terminal value at this point is determined by the following formula:

\[
TV = \frac{\text{Free Cash Flow for the Next Period After Termination Date (FCF)}}{\text{Required Discount Rate} (r)}
\]

This model is similar to the constant growth model. The model says that the terminal value in year five is equal to the capitalized value of the free cash flows for year six. The denominator, however, is the required discount rate rather than the discount rate minus the growth rate that applies in the constant growth model. This model is illustrated as follows:
Example (8). Assume that in Example (7) the estimate of earnings for year six is $1,050,000. This is the year after the year in which the industry is expected to settle into competitive equilibrium. The discount rate is still 15%. The TV for year five is therefore $7 million computed as follows:

\[
TV = \frac{FCF \text{ for the Next Period After Termination Date}}{r}
\]

\[
TV = \frac{1,050,000}{.15}
\]

\[
TV = 7,000,000
\]

The present value of this $7 million terminal value is computed as follows:

\[
\frac{1}{(1 + r)^n} (TV_n)
\]

\[
\frac{1}{(1 + .15)^5} ($7,000,000)
\]

\[(.497)($7,000,000) = 3,479,000\]

This process also involves two discounting steps. First, the free cash flows for the period after the termination date are discounted at the cost of capital to determine the terminal value. Second, the terminal value is discounted at the cost of capital to determine the present value of the terminal value.

Professor Rappaport states that the competitive equilibrium model, which he refers to as the perpetuity method, is generally the best method to use in estimating terminal value:

The perpetuity method for estimating residual value is based on . . . competitive dynamics. It is essentially based on the assumption that a company that is able to generate returns above the cost of capital (i.e., achieve excess returns) will eventually attract competitors, whose entry into the business will drive returns down to the minimum acceptable or cost of capital rate. Specifically, the perpetuity method assumes that after the forecast period, the business will earn, on average, the cost of capital on new investments. Another way of expressing this idea is to say that after the forecast period, the business will invest, on average, in strategies whose net present value is zero.

Once the rate of return has been driven down to the cost of capital rate, period-by-period differences in future cash flows do not alter the value of the business. Therefore, these future flows can be treated as if they were a

199. See supra part III.A (providing the discount factor).
'perpetuity' or an infinite stream of identical cash flows.\textsuperscript{200}

There are several differences between the constant growth model and the competitive equilibrium model. The numerator in the constant growth model is the FCF for the terminal year, whereas the numerator in the competitive equilibrium model is the FCF for the year after the terminal year.\textsuperscript{201} Thus, the numerator in the competitive equilibrium model is larger. The denominator in the constant growth model is the cost of capital minus the growth rate, whereas the denominator in the competitive equilibrium model is the cost of capital. Thus, the denominator in the competitive equilibrium model is larger.

In most cases, the competitive equilibrium model will produce a smaller terminal value than the constant growth model. This smaller terminal value results because the value-depressing effect of the larger denominator in the competitive equilibrium model is likely to more than offset the value-increasing effect of the larger numerator in this model. This can be seen by comparing Examples (7) and (8). In Example (7), which illustrates the constant growth model, the FCF for the terminal year is $1,000,000. In Example (8), which illustrates the competitive equilibrium model, the FCF for the year after the terminal year is $1,050,000, $50,000 larger than that in the constant growth model. The denominator in the constant growth model is 10\%, which equals the cost of capital of 15\% minus the growth rate of 5\%. The denominator in the competitive equilibrium model is 15\%, the cost of capital. Thus, the denominator for the competitive equilibrium model is also larger.

The terminal value determined by the constant growth model is $10 million, whereas the terminal value determined by the competitive equilibrium model is only $7 million. Thus, the value-depressing effect of the larger denominator in the competitive equilibrium model significantly offsets the value-increasing effect of the larger numerator in that model. For the competitive equilibrium model to produce the same $10 million estimate of terminal value predicted by the constant growth model, the FCF for the year after the terminal year would have to be $15 million. This is 50\% more than the FCF for the terminal year, an unrealistic assumption. The point is that the estimate of terminal value can vary significantly under the constant growth model and the competitive equilibrium model. In virtually every case, the constant growth model will produce a higher terminal value because there is an expected growth in the free cash flows. This expected growth adds value.

The difference in the results from the constant growth model and the competitive equilibrium model was also illustrated in the Technicolor case.\textsuperscript{202} The court explained that Professor Rappaport, the expert for Technicolor, used the competitive equilibrium model. Mr. Torkelsen, the expert for the plaintiff shareholders, used the constant growth model:

To establish residual value Rappaport capitalizes a constant (last forecasted year) cash flow; he assumes no new value creation beyond the forecast period (but nevertheless much of his total value is attributed to the

\textsuperscript{200} RAPPAPORT, supra note 6, at 60-61.
\textsuperscript{201} The terminal year is the year the industry is projected to settle into competitive equilibrium.
residual value). In creating his estimation of residual value Torkelsen, on the other hand, increases the last forecasted year's net cash flows by 5% each year (for inflation) into infinity, before capitalizing those flows. The result—and this is the practical gist of this theoretical difference between the experts—is that Mr. Torkelsen assumes that Technicolor net profits (along with all other aspects of its cash flow) and its value will increase every year in perpetuity, while Professor Rappaport assumes there will come a time when, while it may make profits, Technicolor will not be increasing in value.203

The court explained the theory behind Professor Rappaport's approach as follows:

The most basic conceptual difference in the two DCF models used is this: Professor Rappaport assumes (and Mr. Torkelsen does not) that for every company its particular set of comparative advantages establish, as of any moment, a future period of same [sic] greater or lesser length during which it will be able to earn rates of return that exceed its cost of capital. Beyond that point, the company (as of the present moment of valuation) can expect to earn no returns in excess of its cost of capital and therefore, beyond that point, no additional shareholder value will be created. Professor Rappaport calls this period during which a company's net returns can be predicted to exceed its costs of capital, the company's "value growth duration," which is a coined term . . . . It is an application of elementary notions of neoclassical economics: profits above the cost of capital in an industry will attract competitors, who will over some time period drive returns down to the point at which returns equal the cost of capital. At that equilibrium point no new competition will be attracted into the field. The leading finance text includes a reference to this concept of a future period beyond which there is no further value created. (citation omitted). The existence of such a point in time does not mean that there is no value attributed to the period beyond that point, but rather that there is no further value growth.204

The court "accept[ed] as sound (as a 'technique . . . generally considered acceptable in the financial community,' Weinberger, 457 A.2d at 713) the methodology of Professor Rappaport."205

4. Estimating Terminal Value by Reference to Price/Earning Ratios

The first rule of thumb suggested by Brealey and Myers is to look to the price/earnings ratio of companies "whose scale, risk, and growth prospects today roughly match those projected [for the target in the year of termination]."206 Once the comparable price/earnings ratio is determined, that ratio is applied to the expected earnings

203. Id. at *34-35.
204. Id. at *34.
205. Id.
206. BREALEY & MYERS, supra note 6, at 65.
of the target for the terminal year to determine the estimated terminal value. This method is illustrated as follows:

_Example (9)._ Assume that in Example (7) the terminal date is at the end of the fifth year, target’s expected earnings (not free cash flow) for year five are $1.1 million, and the price/earnings ratio of comparable companies is ten. As a consequence, the estimated terminal value of the target at the end of year five is $11 million, computed as follows:

$$10 \times \$1.1 \text{ million} = \$11 \text{ million}$$

The present value of this $11 million terminal value is computed as follows:

$$\frac{1}{(1 + r)^n} (TV_n)$$

$$\frac{1}{(1 + .15)^5} (\$11,000,000)$$

$$(.497) (\$11,000,000) = \$5,467,000$$

Professor Rappaport points out several problems with the use of price/earnings ratios. Rappaport states that it is inherently inconsistent to commingle cash flows during the forecast period with the accounting concept of earnings for the post-forecast period.\(^{207}\) He further points out that the price/earnings approach does not “take into account whether the business can be expected to invest at, below, or above the cost of capital in the post-forecast period.”\(^{208}\) Finally, he points out that there are “no reliable models for accurately forecasting future price/earnings ratios.”\(^{209}\) The price/earnings method is one of the elements used in the comparable companies technique for valuing a target on the basis of present (not future) comparable companies. This can be seen, for example, in Wasserstein/Shearson’s valuation of Warner.\(^{210}\)

5. **Estimating Terminal Value by Reference to Market/Book Ratios**

The second rule of thumb discussed by Brealey and Myers is to use the market/book ratio, which is the ratio of stock price to book value per share. This method involves determining the market/book ratios of companies that are comparable today to the target’s expected composition on the termination date. That ratio is then applied to the target’s expected book value on the termination date. This method is illustrated as follows:

_Example (10)._ Assume that in Example (7) the terminal date is at the end of

---

\(^{207}\) RAPPAポート, _supra_ note 6, at 63.

\(^{208}\) Id.

\(^{209}\) Id.

\(^{210}\) See _infra_ Appendix C, at C-5.
the fifth year, the target’s expected book value is $5 million, and the market book ratio of comparable companies is 2. In such a case the expected terminal value of the target is $10 million (i.e., 2 times $5 million of book value). The present value of this $10 million terminal value is computed as follows:

\[
\frac{1}{(1 + r)^n} \times TV_n
\]

\[
\frac{1}{(1 + .15)^5} \times ($10,000,000)
\]

\[(.497)($10,000,000) = $4,970,000\]

Professor Rappaport points out that conceptual problems with the price/earnings method also exist with the market/book method.211 This market/book method is one of the elements used in applying the comparable companies technique of valuing a target on the basis of present comparable companies.212

6. Summary of the Results Under the Four Methods

The following is a summary of the estimated terminal values and the present value of the terminal values under each of the four methods of determining the terminal value:

<table>
<thead>
<tr>
<th>Method</th>
<th>Terminal Value</th>
<th>Present Value of Terminal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Growth Model</td>
<td>$10,000,000</td>
<td>$4,970,000</td>
</tr>
<tr>
<td>Competitive Equilibrium Model</td>
<td>$7,000,000</td>
<td>$3,479,000</td>
</tr>
<tr>
<td>Price/Earnings Rule of Thumb</td>
<td>$11,000,000</td>
<td>$5,467,000</td>
</tr>
<tr>
<td>Market/Book Rule of Thumb</td>
<td>$10,000,000</td>
<td>$4,970,000</td>
</tr>
</tbody>
</table>

Brealey and Myers explain that although there is no best method for determining terminal value, the most weight should generally be put on the competitive equilibrium method, which sets the terminal date at the point at which positive net present value disappears.213 They also state that the competitive equilibrium method “forces managers to remember that sooner or later competition catches up,” and it is not unusual for the various methods to produce a broad range of estimates of terminal value.214 In summary, they explain, “[d]iscounted cash flow formulas only estimate market value, and the estimates change as forecasts and assumptions change. Managers cannot know market value until an actual transaction takes place.”215

211. RAPPAPORT, supra note 6, at 63-64.
212. See infra Appendix C, at C-5.
213. BREALEY & MYERS, supra note 6, at 67.
214. Id.
215. Id.

Both Lazard, in its opinion to the Warner board, and Wasserstein/Shearson, in their opinion to the Time board, estimated the terminal value of Warner by multiplying the expected cash flow for the terminal period by a factor presumably representing a likely multiple of cash flows for comparable companies. In its presentation to Warner’s board, Lazard used multiples of nine, ten, and eleven times estimated cash flows predicting a range of terminal values for Warner. Linguistically, Lazard, in valuing Warner’s various business segments, used different multiples for each segment. In their presentation to Time’s board, Wasserstein/Shearson used multiples of ten, eleven, twelve, and thirteen in estimating the terminal value of Warner’s film entertainment segment.

J. Use of Formulas for Measuring an Infinite Series of Free Cash Flows

Weston et al. provide formulas for measuring and discounting an expected infinite series of free cash flows. The method requires neither a projection of free cash flows for a horizon period nor the estimation of a terminal value. Rather, it only requires a determination of the free cash flows for either the present year or for both the present year and the next year.

They also provide formulas for each of the following assumptions about the growth of free cash flows: (1) the no growth case, (2) the constant growth case, (3) the supernormal growth followed by no growth case, and (4) the supernormal growth followed by constant growth case.

The formula for the no growth case is similar to the formula used above for determining the terminal value under the competitive equilibrium model. Under this method the present value of an investment that expects no growth in free cash flows is determined by the following formula:

\[
\frac{\text{Free Cash Flow for Current Period (FCF)}}{\text{Cost of Capital (r)}}
\]

The formula for the Weston constant growth case is similar to the formula used previously in computing the terminal value under the constant growth case. The formulas for supernormal growth followed by either no growth or constant growth are beyond the scope of this article.

216. See infra Appendix D, at D-4.
217. See infra Appendix C, at C-3.
218. WESTON ET AL., MERGERS, supra note 2, at 132-63.
219. Id. at 146.
220. Id. at 147.
221. Id.
222. Id. at 148.
223. See supra part V.1.3 (explaining the use of the competitive equilibrium model).
224. See supra part V.1.2 (discussing the constant growth model).
Uncertainty is inherent in the preparation of a cash flow forecast. Therefore, there is uncertainty in computing a single NPV of a project. As Professor Shapiro explains:

"Often . . . [the] single [NPV] number hides a great deal of information about the riskiness of the proposed project: Because the future is unknowable, it is evident that today's estimates of future project prices, costs, and volumes are going to be wrong. It is natural, therefore, for decision makers to want to study, in advance, how potential estimation errors will affect the project NPV." \(^{225}\)

The effect of potential estimation errors may be analyzed through sensitivity analysis. This can involve the preparation of cash flow forecasts on the basis of three possible outcomes: (1) expected, (2) pessimistic, and (3) optimistic. \(^{226}\) Each outcome could be prepared on the basis of different variables in the cash flow projections, such as size, market share, variable costs, sales growth, etc. \(^{227}\) For example, a range of NPVs for a project could be determined by setting each variable at its pessimistic, expected, and optimistic value, while holding all other variables equal to their expected values. \(^{228}\) The following matrix illustrates the projected NPV ranges for various cash flow projections that are determined on the basis of assumptions about market share, variable cost and sales growth.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Cash Flow Range</th>
<th>NPV Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pessimistic</td>
<td>Expected</td>
</tr>
<tr>
<td>(1) Market Share</td>
<td>$9 million</td>
<td>$10 million</td>
</tr>
<tr>
<td>(2) Variable Cost</td>
<td>$3.5 million</td>
<td>$2.5 million</td>
</tr>
<tr>
<td>(3) Sales Growth</td>
<td>8%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Also, a possible range of NPVs can be determined on the basis of a sensitivity analysis that relates a range of assumptions concerning one or more cash flow variables, such as sales growth, to a range of possible costs of capital. The following matrix demonstrates such a sensitivity analysis. \(^{229}\)

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225. SHAPIRO, supra note 6, at 250.
226. BREALEY & MYERS, supra note 6, at 216-18.
227. Id. at 217.
228. Id.
229. The matrix is based on RAPPAPORT, supra note 6, at 236-37.
The expected NPV is $25 million when the cash flows are determined using the expected sales growth and the cash flows are discounted at the expected cost of capital of 13%. A similar matrix can be developed that relates a range of assumptions concerning two different cash flow variables.

This type of sensitivity analysis was used by both Wasserstein/Shearson and Lazard in valuing Warner. Wasserstein/Shearson used three different discount rates and four different estimates of terminal value in valuing Warner’s film entertainment business. Lazard used five different discount rates and three different estimates of terminal value in determining the present value of Warner’s consolidated unleveraged free cash flow.

L. Summary

Part V has focused on the numerator in the DCF model for computing the net present value (NPV) of a project—the determination of both the periodic and terminal free cash flows (FCF):

\[
NPV = \text{Initial Cost (IC)} + \frac{FCF_n}{(1 + r)^n}
\]

The primary reason for an overstatement of NPV is an over-optimistic FCF projection. Acquiring firms should avoid the “winner’s curse” by ensuring that their FCF projections are based on realistic assumptions and that a positive NPV reflects real economic rents. This Article now addresses the denominator in the NPV formula—the computation of the cost of capital (r).

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230. See infra Appendix C, at C-3.
231. See infra Appendix D, at D-4.
232. See supra part III.E.2 (discussing the difficulty of realizing economic rents).
VI. USE OF THE CAPITAL ASSET PRICING MODEL (CAPM) IN DETERMINING THE APPROPRIATE DISCOUNT RATE

A. Introduction

The discount rate \( (r) \) in the NPV formula is the cost of capital, the minimum rate of return necessary to induce an acquiring firm to acquire the target. It is the rate at which the free cash flows and terminal value of an acquisition candidate are discounted to present value. This part examines the capital asset pricing model (CAPM), which provides a method for determining the discount rate. There are two other methods for determining the discount rate: the arbitrage pricing theory (APT)\textsuperscript{233} and the weighted average cost of capital (WACC)\textsuperscript{234}

This Article will separate the financing decision from the investment decision and use CAPM to determine the discount rate (cost of capital) on the basis of the assumption of all-equity financing. Under this method, the benefits or costs associated with the financing decision are separately determined and discounted to present value at the cost of capital. A similar approach is used with APT.\textsuperscript{235}

WACC provides a method for determining the cost of a firm’s debt and equity financing. In determining the discount rate, WACC directly takes into account the financing decision and, therefore, the impact of the interest deduction on the debt. Thus, under WACC, the benefit of the interest deduction is not separately determined and discounted, but rather is reflected in a lower cost of capital. In using WACC to determine the cost of capital, as when using CAPM or APT, the cash flows are projected on the basis of an all-equity financing.

Copeland et al. recommend the use of either CAPM or APT for determining the cost of equity capital used in WACC.\textsuperscript{236} Thus, CAPM and APT have a role even in the use of WACC. Copeland et al. further state that although both the CAPM and APT approaches have problems “associated with their application . . . , [both] are theoretically correct; they are risk adjusted and account for inflation.”\textsuperscript{237} They also assert that other methods for determining the cost of equity, such as the dividend yield model and the earnings to price ratio model, are “conceptually flawed.”\textsuperscript{238}

Both CAPM and APT are based on the Efficient Capital Markets Hypothesis.\textsuperscript{239} This hypothesis, which this Article only outlines, posits that if investors have easy access to information about a security, the price of that security will properly reflect that information.\textsuperscript{240} Consequently, “[i]f capital markets are efficient, then purchase or sale of any security at the prevailing market price is never a positive NPV transac-

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\textsuperscript{233} See infra part VII (briefly introducing APT).

\textsuperscript{234} See infra part IX (discussing the use of WACC).

\textsuperscript{235} See also infra part VIII (discussing the interaction between the investment and financing decisions).

\textsuperscript{236} COPELAND ET AL., VALUATION, supra note 6, at 192.

\textsuperscript{237} Id.

\textsuperscript{238} Id.

\textsuperscript{239} See, e.g., Fama, Efficient Capital Markets II, supra note 17, at 1589-99 (discussing APT’s market efficiency theory basis).

\textsuperscript{240} BREaley & Myers, supra note 6, at 290.
— the price of the security will equal the present value of the expected cash flows discounted at the appropriate cost of capital. As Brealey and Myers explain, “the capital asset pricing model boils down to the statement that the market portfolio (i.e., a portfolio of all stocks) is efficient.” Thus, the purchase of the market portfolio would not create a positive NPV.

To understand the principles behind CAPM it is first necessary to consider the concepts of (1) systematic and unsystematic risk, (2) the measure of market risk (beta), and (3) the market risk premium. Professors Fama and French have recently criticized CAPM, and others have come to the defense of CAPM.

CAPM gives the rate of return \( r \) for a particular investment by a formula that takes into account the following variables: (1) the risk-free rate of return \( r_f \), (2) the market risk premium \( r_m - r_f \), and (3) the beta (\( \beta \)) of the investment. The formula is as follows:

\[
    r = r_f + \beta (r_m - r_f)
\]

This CAPM formula posits that the required rate of return for an investment is equal to the risk-free rate plus an additional amount equal to the beta for the investment multiplied by the market risk premium. The cost of capital determined under CAPM looks to the returns available to investors in the capital markets on other investments with similar risk patterns.

**B. Systematic and Unsystematic Risk**

There are two types of risk inherent in investments: systematic risk and unsystematic risk. Risk in either context means that future returns are uncertain. Systematic risk is a function of broad macroeconomic conditions that affect the prices of all assets. Thus, there is systematic risk in holding any asset. Conversely, unsystematic risk is a function of the characteristics associated with a particular asset as opposed to

241. Id. at 289.
242. Id. at 164.
243. See infra part VLE (introducing CAPM).
244. See infra part VLB (discussing systematic and unsystematic risk).
245. See infra part VLC (discussing beta).
246. See infra part VLD (discussing risk-free rate and market risk premium).
247. See infra part VLG (discussing the Fama and French critique of CAPM and the responses).
248. The market risk premium is the difference between the expected return on the market \( r_m \) and the risk-free rate \( r_f \).
249. See infra part VLE (explaining CAPM more fully).
250. BREALEY & MYERS, supra note 6, at 190-97.
251. SHAPIRO, supra note 6, at 276; see Jeffrey N. Gordon & Lewis A. Kornhauser, Efficient Markets, Costly Information and Securities Research, 60 N.Y.U. L. REV. 761, 775-86 (1985) (discussing much of the literature dealing with CAPM, including some of the critiques of the concept) [hereinafter Gordon & Kornhauser, Efficient Markets].
252. BREALEY & MYERS, supra note 6, at 149; see also GILSON & BLACK, LAW AND FINANCE, supra note 6, at 81-100.
253. BREALEY & MYERS, supra note 6, at 137.
broad market factors. For example, risk factors such as the rate of growth of GNP or interest rate levels "systematically affect all firms in the economy to a greater or lesser extent." Comparatively, risk factors such as possible changes in consumer tastes, new product developments, and changes in prices of raw materials affect particular firms and, therefore, are referred to as unsystematic risks.

Diversification can reduce or eliminate unsystematic risk by balancing the losing stocks with winning stocks; such risk is often referred to as diversifiable risk, unique risk, residual risk, or specific risk. The specific risk of holding stock in a particular company can be diversified away by investing in stocks of other companies, possibly in other industries. Diversification cannot eliminate systematic risk, however. Systematic risk is often referred to as undiversifiable risk or market risk. Since market risk results from broad macroeconomic factors threatening most businesses, stocks have a tendency to move in the same direction.

Brealey and Myers elaborate on the concepts of unique (or unsystematic) risk and market (or systematic) risk:

If you only have a single stock, unique risk is very important, but once you have a portfolio of 20 or more stocks, diversification has done the bulk of its work. For a reasonably well-diversified portfolio, only market risk matters. Therefore, the predominant source of uncertainty for a diversified investor is that the market will rise or plummet, carrying the investor’s portfolio with it.

The following graph illustrates total risk as a function of both systematic and unsystematic risk and the reduction of unsystematic risk through diversification.

![Graph A](image-url)
This graph illustrates that the greater the diversification (represented on the horizontal axis), the less the total risk (measured on the vertical axis). Unsystematic risk is reduced by diversification; systematic risk is not. Diversification reduces unsystematic risk because the prices of different stocks move differently, the price movements are less than perfectly correlated, and the decline in the prices of some stocks are offset by increases in the prices of others. Brealey and Myers explain that “[w]ith more securities, and therefore better diversification, portfolio risk declines until all unique risk is eliminated and only the bedrock of market risk remains.”

C. Beta: The Measure of Market Risk

1. Introduction

Beta is the measure of the sensitivity of a security’s return to market movements. Thus, it is a measure of the systematic risk of a security. Beta indicates what the likely move for the particular stock will be considering a given move in the market—how sensitive a particular stock is to a move in the market. For a stock with a beta of one, a 10% market rise would, on average, lead to a 10% rise in the price of the stock. For a stock with a beta of 1.5, a market rise of 10% would lead on average to a 15% rise in the price of the stock. Thus, beta “measures the marginal contribution of a stock to the risk of the market portfolio.” Beta also “gauges the tendency of the return on a security to move in parallel with the return of the stock market as a whole.”

The beta for a particular stock is calculated by running a linear regression between past returns for that stock and past returns of a market index, such as the Standard and Poor’s 500. Betas are generally computed on the basis of past data (i.e., ex post). Techniques exist, however, for estimating ex ante, or future, betas.

2. Statistical Derivation of Beta

Beta for a particular stock is determined by dividing (1) the covariance between returns on the particular stock, and returns on the market portfolio; by (2) the variance of the market portfolio. The variance of the market portfolio indicates the potential for deviation of the market’s return from the market’s expected return. The market’s expected return (i.e., the mean or average return) is the profit expected from
the market as a rate of return for a specified period. 272

The computation of the market's expected return can be illustrated as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>40</td>
<td>40%</td>
<td>6%</td>
</tr>
<tr>
<td>10%</td>
<td>20</td>
<td>20%</td>
<td>2%</td>
</tr>
<tr>
<td>0%</td>
<td>40</td>
<td>40%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td>8% Expected Return</td>
</tr>
</tbody>
</table>

The probability of the outcome is determined by analyzing historical data concerning the market's possible returns and the frequency of those returns. 273 In summary, the expected return is the weighted average return, found by multiplying each possible return by the corresponding probability and summing the results.

In computing variance, the market's expected return is subtracted from the market's actual return for a particular period or periods. 274 The resulting amount is then squared, and this amount is then multiplied by the probability of such occurrence to get the variance. The formula can be written as follows: 275

\[
\text{Variance } (r_m) = \frac{\{\text{Actual Rate of Return for Particular Period } (r)\}^2 - \{\text{Expected Market Return } (r)\}}{\{\text{Probability of Realizing the Actual Rate }\}}
\]

272. Id.
273. BREALEY & MYERS, supra note 6, at 132-33.
274. Id.
275. Id.
Thus, the three-step process for computing variance can be illustrated as follows. The first step is to subtract the market’s expected return from the actual return to compute the deviation from the mean:

<table>
<thead>
<tr>
<th>ACTUAL RETURN FOR PARTICULAR PERIODS</th>
<th>EXPECTED RETURNS</th>
<th>DEVIATION FROM MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>-8</td>
</tr>
</tbody>
</table>

The second step is to square the deviations from the mean:

<table>
<thead>
<tr>
<th>Squared Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 x 7 = 49</td>
</tr>
<tr>
<td>2 x 2 = 4</td>
</tr>
<tr>
<td>-8 x -8 = 64</td>
</tr>
</tbody>
</table>

The third step is to multiply the squared deviation by the probability of occurrence of the deviation and sum to get the variance:

<table>
<thead>
<tr>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squared Deviation Times Probability</td>
</tr>
<tr>
<td>49 x .40 = 19.60</td>
</tr>
<tr>
<td>4 x .20 = .8</td>
</tr>
<tr>
<td>64 x .40 = 25.60</td>
</tr>
<tr>
<td>46 Variance</td>
</tr>
</tbody>
</table>
The market's expected return and variance "provide information about the probability distribution associated with the ... portfolio of stocks constituting [the market]."\textsuperscript{276}

The covariance is "a measure of the way in which two random variables move in relation to each other—the way they covary."\textsuperscript{277} The covariance between two stocks is determined by multiplying the correlation coefficient\textsuperscript{278} (COR\textsubscript{1,2}) for the two stocks by the standard deviations (\( \sigma \)) of the two stocks. The correlation coefficient measures the degree to which the movements of the prices of two stocks are correlated—move together.\textsuperscript{279} The standard deviation is the positive square root of the variance of the stock.\textsuperscript{280} Thus, the standard deviation of stock one is written as:\textsuperscript{281}

\[
\text{Standard Deviation (} \sigma_1 \text{) of Variance (} r_i \text{)} = \sqrt{\text{VARIANCE (} r_i \text{)}}
\]

Continuing this illustration, the square root of the variance of 46 is 6.7. Thus, the standard deviation is 6.7. This means that for normal distributions,\textsuperscript{282} \( 68.27\% \) of the actual returns are within one standard deviation (i.e., 6.7) on either side of the expected return; 95.45\% of the actual returns are within two such standard deviations, and 99.73\% of the actual returns are within three such standard deviations.

The covariance formula can be written as:

\[
\text{Covariance between Stock 1 and 2} = \sigma_{1,2} = \text{COR}_{1,2}\sigma_1\sigma_2
\]

This formula posits that the covariance between stocks one and two (\( \sigma_{1,2} \)) is equal to the correlation coefficient between stocks one and two (\( \text{COR}_{1,2} \)), multiplied by their standard deviations (\( \sigma_1, \sigma_2 \)). If the covariance is positive, the prices of the two stocks move in the same direction.\textsuperscript{283} If the covariance is negative, the prices move in opposite directions.\textsuperscript{284} The covariance between the returns on a particular stock and the returns on the market portfolio measures the way the expected returns on the stock move in relation to the expected returns of the market.

The beta of stock one is derived by dividing (1) the covariance between (a) the returns on stock one, with (b) the returns on the market portfolio (\( \text{COR}_{1,m}\sigma_1\sigma_m \)), by (2) the variance of the market portfolio (\( \sigma_m^2 \)). The formula can be written as:\textsuperscript{285}

\[
\text{Beta of Stock 1} = \frac{\text{COR}_{1,m}\sigma_1\sigma_m}{\sigma_m^2}
\]

\textsuperscript{276} HAU
gen, supra note 6, at 56.

\textsuperscript{277} BR\text{ealey \\& Myers, supra note 6, at 140.}

\textsuperscript{278} Id. The formula for computing the correlation coefficient is not set forth here.

\textsuperscript{279} HAU

gen, supra note 6, at 52-55.

\textsuperscript{280} BR\text{ealey \\& Myers, supra note 6, at 134.}

\textsuperscript{281} COPE\text{land \\& Weston, supra note 6, at 150.}

\textsuperscript{282} A normal distribution will produce a bell-shaped curve. See HAU
gen, supra note 6, at 198.

\textsuperscript{283} For example, the covariance between the stock prices of two computer firms will likely be positive.

\textsuperscript{284} COPE\text{land \\& Weston, supra note 6, at 156.}

\textsuperscript{285} See BR\text{ealey \\& Myers, supra note 6, at 145.}
This formula indicates that the beta of stock one is equal to (1) the product of (a) the correlation coefficient between stock one and the market (COR₁ₘ), (b) the standard deviation of returns on stock one (σ₁), and (c) the standard deviation of returns on the market (σₘ); divided by (2) the variance of the market (σₘ²).

Thus, the beta of the market portfolio is equal to one, because the "covariance of the market portfolio with itself is identical to the variance of the market portfolio." This can be established as follows:

\[
\frac{\text{COR}_{m,m} \sigma_m \sigma_m}{\sigma_m^2} = \frac{\sigma_m}{\sigma_m^2} = 1
\]

Comparatively, the beta of a risk-free asset, such as a Treasury bill, is equal to zero "because its covariance with the market portfolio is zero,"—there is no correlation between the price of Treasury bills and the price of the market. This can be established as follows (T represents Treasury bills):

\[
\frac{\text{COR}_{T,m} \sigma_T \sigma_m}{\sigma_m^2} = \frac{0 \sigma_T \sigma_m}{\sigma_m^2} = 0
\]

---

286. COPELAND & WESTON, supra note 6, at 198.
287. Id. at 198.
The beta of a particular stock, which can be negative, zero or positive, is an indicator of the degree to which the price of the stock changes in relation to changes in the market.  

3. Further Elaboration on Beta

Because the market has a beta of one, a high beta stock is one with a beta greater than one and a low beta stock is one with a beta less than one. A high beta stock tends to move up by a greater percentage than the market moves up and tends to move down by a greater percentage than the market moves down. Such stocks thereby tend to “amplify the overall movements of the market.” However, “stocks with a beta between zero and one tend to move in the same direction as the market, but not as far.” Stocks with negative betas tend to move in the opposite direction of the market.

The following graph illustrates a stock that has a beta, or sensitivity to the market, of two:

A diversified portfolio of stocks with high betas has more risk than a diversified portfolio of stocks with low betas. As Brealey and Myers point out, “the risk of a well diversified portfolio is proportional to the portfolio’s beta, which equals the average betas of the securities included in the portfolio.” They further emphasize that “[t]he risk of a well diversified portfolio depends on the average beta of the securities included in the portfolio.” Portfolio risk declines with diversification until the point

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288. Haugen, supra note 6, at 58.
289. See Brealey & Myers, supra note 6, at 143.
290. Id.
291. Id. at 144.
292. Id.
293. Id. at 145.
294. Brealey & Myers, supra note 6, at 144.
where all unique or specific risk is eliminated and only the "bedrock of market risk remains."295

Betas estimate the amount by which investors expect the price of a particular stock to change as a result of a change in the market. A stock with a beta greater than one, such as Tandem Computer,296 is generally sensitive to movements in the market, whereas a stock with a low beta, such as AT&T,297 is less sensitive to market moves. This is because the cash flows of AT&T are more certain than the cash flows of Tandem Computer; that is, there is more risk associated with Tandem’s cash flows.298

Professor Shapiro explains that “[s]tocks with betas greater than one are often called aggressive stocks because they go up faster than the market in a ‘bull’ (rising) market but fall faster in a ‘bear’ (declining) market.”299 On the other hand, stocks with betas lower than one are referred to as “defensive stocks” and “tend to fluctuate less than the market.”300

Investment services often estimate the betas of various companies.301 For example, Merrill Lynch publishes a Beta Book.302 Also, analysis of industry-specific betas allows for the estimation of betas for particular projects or divisions of larger firms.303 In determining betas firms may have to adjust for financial risks (i.e., leverage) that they employ.304

D. Risk-Free Rate and Market Risk Premium

In addition to determining beta, in applying CAPM it is necessary to determine the risk-free rate and the market risk premium. The return on Treasury bills measures the risk-free rate of return.305 The rate is risk-free because there is no safer investment and the risk of default is essentially nonexistent. In comparison, a well diversified portfolio of common stocks offers the market return. The market return has to be greater over time than the return on Treasury bills, otherwise no one would invest in common stocks. Because the risk of holding common stocks is greater than the risk of holding Treasury bills, the reward for holding common stock must be greater than the reward for holding Treasury bills.

Historically the market risk premium—the excess of the return on a diversified portfolio of common stocks over the Treasury bill rate—has been 8.4%.306 Thus, if the

295. Id.
296. Id. at 163.
297. Id.
298. Id. at 162.
299. SHAPIRO, supra note 6, at 120.
300. Id.
301. BREALEY & MYERS, supra note 6, at 186.
302. See also the beta estimates provided by Wilshire Associates, BARRA, and Bloomberg. For some rules of thumb for determining beta, see COPELAND ET AL., VALUATION, supra note 6, at 197.
303. BREALEY & MYERS, supra note 6, at 189.
304. See infra part V.F (discussing the difference between asset betas, which are unleveraged, and equity betas, which account for a firm’s leverage).
305. BREALEY & MYERS, supra note 6, at 161. But see COPELAND ET AL., VALUATION, supra note 6, at 192 (suggesting that the risk-free rate should be the rate on ten-year Treasury bonds).
306. BREALEY & MYERS, supra note 6, at 131; see also COPELAND ET AL., VALUATION, supra note 6, at
Treasury bill rate is presently 7%, the expected rate of return (including dividends and capital gains) from holding a well diversified portfolio of common stocks should be approximately 15.4%.

The algebraic formula for market risk premium, where $r_m$ means the market return and $r_f$ means the risk-free rate on Treasury bills, is:

$$\text{Market Risk Premium} = (r_m - r_f)$$

E. The Capital Asset Pricing Model

The elements of CAPM are (1) the beta for the particular stock; (2) the risk-free rate ($r_f$), and (3) the market risk premium ($r_m - r_f$). A market portfolio of common stocks has a beta of one because the “covariance of the market with itself is identical to the variance of the market portfolio.” The expected risk premium of holding a market portfolio is equal to the market risk premium ($r_m - r_f$), and the expected return of holding the market portfolio is equal to the sum of the risk free rate ($r_f$), and the market risk premium ($r_m - r_f$). Thus, there is a one-to-one relationship between the beta of a portfolio of common stocks and the expected return of such a portfolio. The following graph illustrates this one-to-one relationship:

This graph shows that the beta of a risk-free Treasury bill, which has an expected return of $r_f$, is zero, and the beta for the market portfolio, which has a return of $r_m$ is one. The graph also illustrates that any security with a beta of less than one can expect

193 (stating that if the rate on ten-year Treasury bonds is taken as the risk free rate, the market risk premium is five to six percent).


308. See supra part VI.C.2; COPELAND & WESTON, supra note 6, at 198.
a return that is less than the market return because such a security is less risky than the market portfolio. Furthermore, any security with a beta greater than one has an expected return that is more volatile than that of the market because that security is more risky than the market portfolio. Thus, the cost of capital for a low beta stock will be less than the cost of capital for the market, and the cost of capital of a high beta stock will be higher than the cost of capital for the market. For example, AT&T, which has a beta of less than one, will have a lower cost of capital than Tandem Computer, which has a beta greater than one.

The line intersecting the risk-free rate on the vertical axis is known as the security market line (SML). The SML illustrates the capital asset pricing model, which provides that “the expected risk premium on each investment is proportional to its beta.”

GRAPH D
SECURITY MARKET LINE

The expected risk premium on a particular stock can, therefore, be written as:310

\[
\text{Expected Risk Premium} = (\text{Beta}) \times (\text{Expected Risk Premium of Market}) \\
= (\text{Beta}) \times (r_m - r_f) \\
= \beta (r_m - r_f)
\]

The expected rate of return (or the discount rate, hurdle rate, or cost of capital) can be derived from the algebraic statement of the CAPM:

\[
\text{Expected Rate of Return} = r = (\text{Risk-Free Rate of Return}) + (\text{Beta}) \times (\text{Market Risk Premium}) \\
r = r_f + \beta (r_m - r_f)
\]

309. BREALEY & MYERS, supra note 6, at 162.
310. Id. at 138.
In a well functioning capital market, "a security cannot sell for an extended period at prices low enough to yield more than the appropriate return indicated on the SML." Such a security would become an attractive investment relative to other securities with similar risk. The demand for the security would cause its price to rise until the expected return fell to the appropriate point on the SML. Conversely, a stock with a price high enough to put its expected return below the appropriate point on the SML would induce investors to sell the stock, thereby driving down the price until the rate of return was appropriately placed on the SML.

Thus, in using CAPM to determine the appropriate discount rate for the acquisition of a target corporation, it is necessary to identify: (1) the risk-free rate (i.e., the rate on Treasury bills), (2) the market risk premium, which over time has been 8.4%, and (3) the beta for the target corporation. Although there is an element of judgment in determining beta, most estimates should be in the same ballpark.

If the target is involved in several lines of business, it is appropriate to determine, using the CAPM, a discount rate for each segment and to discount to present value, using the DCF model, the estimated free cash flows and terminal value of each line of business. In the words of Brealey and Myers, "[t]he true cost of capital depends on the use to which the capital is put." This is basically the method that was followed in valuing Warner's different business segments, both by Wasserstein/Shearson in their opinion to Time's board and by Lazard in its opinion to Warner's board. Both firms used a range of discount rates for each segment, but neither report indicates whether the discount rates were determined by using CAPM.

Some companies use different discount rates depending on the type of investment being analyzed. For example, an investment that expands the firm's core business may be analyzed with a discount rate equal to the firm's cost of capital, determined using WACC. A more risky venture may be analyzed with a higher discount rate, and a less risky venture may have a lower rate.

CAPM was used by the Delaware Court of Chancery in the appraisal decision in Cede & Co. v. Technicolor, Inc., where the court said, "[T]he CAPM methodology is certainly one of the principle [sic] 'techniques or methods . . . generally considered acceptable [for estimating the cost of equity capital component of a discounted cash flow modeling] in the financial community . . . .'"
F. Asset and Equity Betas

A firm's cost of capital is a function of its business risk—the risk associated with the firm's investments or assets.\textsuperscript{320} In a firm with no debt, the common shareholders face only this business risk. But if the firm carries debt (i.e., is leveraged), the common shareholders also face financial risk: the risk that the firm will not be able to service the debt and therefore will go into bankruptcy.\textsuperscript{321} Obviously, the more leverage the greater the risk to the common shareholders. Financial risk, however, does not affect business risk; leverage does not affect the firm's assets.\textsuperscript{322}

In determining the beta of a firm's equity, the beta must be adjusted to reflect the financial risk (i.e., leverage) employed by the particular company. A firm's leverage controls its level of financial risk and its beta. As a firm's leverage increases, its financial risk and beta correspondingly increase.\textsuperscript{323} A firm's asset beta would apply if the firm did not employ any leverage. Therefore, the asset beta assumes a firm incurs business risk, but not financial risk.

Brealey and Myers explain that because low risk is generally associated with debt of large blue-chip firms, financial analysts generally assume that the beta of the debt of such firms is zero. When the firm's debt beta is zero, its debt is not sensitive to moves in the equity market.\textsuperscript{324} This concept of stability is intuitively appealing. Debt holders in firms that are not highly leveraged can be fairly certain that they will be paid regardless of market shifts. Assuming Brealey and Myers are correct, the value of the debt is not affected by market swings. Brealey and Myers explain, however, that stability may not occur in periods of volatile interest rates such as the early 1980s, when betas on corporate bonds were as high as .3 to .4.\textsuperscript{325} In a highly leveraged firm, it can be expected that the value of the debt will change with changes in the market. However, regardless of whether the value of a firm's debt fluctuates or remains constant, investing in the firm's debt will be less risky than holding equity in the same firm.

If the beta of a firm's debt is zero, the beta of its stock will equal the firm's asset beta. The asset beta is equal to the sum of the weighted average of the beta of the firm's debt ($D$) and the beta of the firm's equity ($E$).\textsuperscript{326} Thus, the firm's asset beta can be expressed algebraically as:

$$\beta_{\text{assets}} = \frac{D}{V} \beta_{\text{debt}} + \frac{E}{V} \beta_{\text{equity}}$$

In this formula $\frac{D}{V}$ is the portion of the firm's value attributable to debt, and $\frac{E}{V}$ is the portion of the value attributable to equity.

\textsuperscript{320} Brealey & Myers, supra note 6, at 189.
\textsuperscript{321} Id.
\textsuperscript{322} Id. at 190.
\textsuperscript{323} Id. at 191-92; see also Copeland & Weston, supra note 6, at 455-60.
\textsuperscript{324} Brealey & Myers, supra note 6, at 191.
\textsuperscript{325} Id.
\textsuperscript{326} Id.
If the beta of the firm’s debt is positive (meaning the debt is sensitive to market moves), the beta for the firm’s equity will exceed the firm’s asset beta because of the financial risk associated with the debt. The weighted averages of the firm’s debt beta and equity beta will equal the firm’s asset beta (i.e., the beta of the nonleveraged or low leveraged firm).

The following illustration helps conceptualize these ideas. Assume that a nonleveraged firm has an asset beta of .8. This is also the firm’s equity beta, as illustrated on the following graph:

This means that without any debt the stock is not very sensitive to market moves—the price of the equity will move both up less and down less than the market.

Now assume that the firm takes on such substantial debt that debt represents 40% of the value of the firm, and the debt has a beta of .2. The firm’s equity beta, which is

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327. Id. at 191-92 (providing the basis for the illustration).
now 1.2, can then be computed algebraically since the only unknown in the formula is the equity beta:

$$\text{Beta of assets} = \frac{D}{V} \text{Beta of debt} + \frac{E}{V} \text{Beta of equity}(x)$$

$$0.8 = (0.4)(0.2) + (0.6)(x)$$

$$0.8 = 0.08 + (0.6)(x)$$

$$\frac{0.8 - 0.08}{0.6} = x$$

$$\frac{0.72}{0.6} = x$$

$$1.2 = x$$

This result is intuitively appealing because it can be expected that, as a firm becomes more and more leveraged, the equity holders will demand a higher return to compensate themselves for the added financial risk.

The relationship between the debt and asset betas in the above case can be diagramed as:
The following principles can be derived by relating the previous diagram to the following formula for determining the asset beta:

\[
\text{Beta of asset} = \frac{D}{V} \text{ Beta of debt} + \frac{E}{V} \text{ Beta of equity}
\]

First, the lower the beta of debt, the lower the beta of equity. If debt has a beta of zero, the beta of equity is equal to the asset beta. Second, the higher the beta for debt, the higher the beta for equity. Third, the asset beta equals the weighted average of the firm's debt and equity betas. Fourth, the firm's asset beta does not change with leverage, as asset betas are not affected by financial risk. This, too, is intuitively appealing because the income stream does not depend on whether the cash flow pays the firm's debt holders or equity holders.

Many industry-wide betas reflect asset betas after removing the effects of financial leverage on beta. Thus, determining the equity beta of a particular firm in the industry requires adjusting the asset beta upward for any leverage utilized.\(^\text{328}\) However, if an equity beta is available, it may be necessary to convert the equity beta into an asset beta, a process referred to as "unlevering."\(^\text{329}\)

In the acquisition context, it is generally appropriate to use asset betas in determining the discount rate for the DCF model. If asset betas are used, any cost or benefits associated with the financing of the acquisition (e.g., tax benefits from interest deductions) should be separately determined for each year and then discounted to present value with the other cash flows.\(^\text{330}\) In certain situations it may be appropriate to use equity betas. For example, the equity beta may be used in analyzing a partial stock investment in a leveraged firm and in determining the cost of equity capital when applying WACC.\(^\text{331}\)

If the asset beta is used to determine the value of a target's assets in the DCF model, the target's outstanding debt is subtracted to ascertain the value of the equity. This technique of valuing the equity—first determining the value of assets with DCF and then subtracting the outstanding debt—was utilized by both Wasserstein/Shearson in its presentation to Time's board\(^\text{332}\) and by Lazard in its presentation to Warner's board.\(^\text{333}\)

**G. Fama and French Critique of CAPM and Responses**

Although CAPM was criticized before 1992, most of the empirical studies similarly concluded that "[s]ecurity returns appear to be linearly related to beta as predicted by the CAPM," and "[t]here is a positive relationship between beta and past returns; that is, higher betas tend to lead to higher returns."\(^\text{334}\) A 1992 empirical examination

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328. *Id.* at 190-92.
329. See SHAPIRO, *infra* note 6, at 263-65 (discussing the unlevering process).
330. See *infra* part VIII (dealing with the interaction between the investment decision and the financing decision).
331. See *infra* part IX (discussing WACC).
332. See *infra* Appendix C, at C-7.
333. See *infra* Appendix D, at D-4.
334. SHAPIRO, *infra* note 6, at 124; see Gordon & Kornhauser, *Efficient Markets*, *infra* note 251, at 775-
of beta conducted by Professors Fama and French seriously questioned the validity of the “basic prediction” of CAPM: “average stock returns are positively related to market betas.” They concluded that beta “does not seem to help explain the cross-section of average stock returns.” They found that the size of a firm’s market equity (ME)—a stock’s price times shares outstanding—and book to market equity (BE/ME) “provide a simple and powerful characterization of the cross-section of average stock returns for the 1963-1990 period.”

Fama and French found that if common stock portfolios are formed on the basis of size alone, “there seems to be evidence for the model’s central prediction: average return is positively related to β.” They pointed out, however, that the betas of portfolios ranked by size are “almost perfectly correlated with size,” but when portfolios are subdivided on the basis of betas, there is a “strong relation between average return and size, but no relationship between average return and β.”

Fama and French formed twelve portfolios of securities on the basis of firm size. The data demonstrates a strong negative relationship between firm size and average return, with average returns falling from 1.64% per month for the portfolio containing the smallest firms to 0.90% for the portfolio containing the largest firms. A strong positive relationship between average returns and beta was also shown, with beta declining from 1.44 for the portfolio containing the smallest firms to 0.90 for the portfolio containing the largest firms. These basic data can be summarized as follows:

<table>
<thead>
<tr>
<th>Portfolios Formed on Size</th>
<th>Portfolio with Smallest Firms</th>
<th>Portfolio with Largest Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Monthly Returns</td>
<td>1.64%</td>
<td>0.90%</td>
</tr>
<tr>
<td>Beta</td>
<td>1.44%</td>
<td>0.90%</td>
</tr>
</tbody>
</table>

Fama and French asserted that although a “simple size sort seems to support the . . . prediction of a positive relation between β and average returns . . . the evidence is muddled by the tight relation between size and the β’s of size portfolios.” They concluded that the “proper inference seems to be that there is a relation between size and average return, but controlling for size there is no relation between β and aver-

76 (discussing much of the literature dealing with CAPM, including some of the critiques of the concept).
335. Fama & French, supra note 18, at 428; cf. Fama, Efficient Capital Markets II, supra note 17, at 1589-99 (providing an overall evaluation of CAPM and APT in the context of market efficiency).
336. Fama & French, supra note 18, at 428.
337. Id. at 429.
338. Id. at 432.
339. Id.
340. Id. at 433.
341. Fama & French, supra note 18, at 433.
342. Id. at 436.
343. Id. at 433.
They also found a strong relationship between average returns and book to market equity.345

In explaining the implications of their findings, Fama and French stated that their “results imply that the performance of managed portfolios . . . can be evaluated by comparing their average returns with the average returns of benchmark portfolios with similar size and BE/ME characteristics” and that the “expected returns from different portfolio strategies can be estimated from the historical average returns with matching size and BE/ME properties.”346 This conclusion may imply that the comparable companies and comparable transactions valuation techniques,347 which consider firms of similar size, are more likely to produce an appropriate valuation of a target than the DCF technique.

Finally, Fama and French stated that “if there is a role for β in average returns, it is likely to be found in a multi-factor model that transforms the flat simple relation between average return and β into a positively sloped conditional relational.”348 Arbitrage pricing theory (APT) is an example of the type of multi-factor model referred to by Fama and French.349

It appears that, notwithstanding the findings of Fama and French, CAPM will still be used as a method of determining an appropriate discount rate. In a critique of the Fama and French study, Professors Roll and Ross conclude that the “idea of a tradeoff between risk and return [which is at the heart of CAPM] is valid and meaningful . . . [and] . . . the empirical findings are not by themselves sufficient to cause rejection of the theory.”350 Roll and Ross also point out that a study by Amihud, Christensen, and Mendelson replicating the Fama and French test, but utilizing more advanced techniques, concludes that “beta is alive and well.”351

Another critique of the Fama and French study, authored by Fisher Black concludes:

If today’s corporations do not face borrowing restrictions, and if the corporation makes its investment decisions to maximize its stock price, the market for corporate assets should be governed by ordinary CAPM. A firm should use discount rates for its investments that depend on their betas in the usual way.352

Finally, in a 1995 article entitled The CAPM Debate, the authors observe:

With academics debating the value of the CAPM, what are companies that now use it in their capital budgeting process to do? Maybe nothing different . . . . [T]he data seem to suggest that those who choose to use the

344. Id.
345. Id. at 440.
346. Fama & French, supra note 18, at 452.
347. See infra part XI.
348. Fama & French, supra note 18, at 449.
349. See infra part VII.
350. Roll & Ross, supra note 19, at 115.
351. Id. at 113-14.
CAPM now despite the academic debate will actually not be getting worthless advice.... [F]or those interested in the longer view, the CAPM still seems to have something to offer.”

The CAPM model is intuitively appealing, and Fama and French did find a relationship between beta and average returns when firms are organized according to size. Adjustments could be made in utilizing CAPM to take account of size differences. Although CAPM will still be used, it can be expected that greater emphasis will be given to both APT and the comparable companies and comparable transaction techniques.

VII. USE OF ARBITRAGE PRICING THEORY (APT) IN DETERMINING THE APPROPRIATE DISCOUNT RATE

CAPM provides a basic economic model for determining the discount rate; the arbitrage pricing theory (APT) provides an alternative, albeit more elaborate economic model. CAPM is a unidimensional model focusing only on the return of the market. On the other hand, APT attempts to account for all of the macroeconomic factors that can affect the price of a particular stock. The expected risk premium for each factor is measured, and then the stock’s sensitivity to each factor is determined. Finally, the expected return is computed. Needless to say, this is an extremely complicated process. As Copeland et al. explain, “APT can be thought of as a multifactor analogue to the CAPM.”

Empirical research generally indicates that no more than three to five common factors affect stock prices and must therefore be taken into account in APT. The four most common factors seem to be unexpected changes in industrial output, unexpected changes in inflation, the difference between the yield on long-term and short-term Treasury bonds, and bond risk premiums. Professor Shapiro explains that “[t]hese variables make intuitive sense as risk factors because unanticipated changes in them systematically affect the value of all assets.”

Brealey and Myers have compared industry discount rates determined under CAPM and APT. The comparison shows, for example, that for railroads the discount rate under both models is the same: 17.9%. For the paper industry, the discount rates...
rate using CAPM (17.1%) is less than that determined using APT (18.4%). For utilities, however, the discount rate using CAPM (13.5%) is higher than the rate determined under APT (11.4%). Commenting on the two methods, Brealey and Myers—who seem to prefer CAPM—state, "[e]ach of these different models of risk and return has its fan club. However, all financial economists agree on two basic ideas: (1) Investors require extra expected returns for taking risk, and (2) they appear to be concerned predominantly with the risk they cannot eliminate by diversification."

VIII. INTERACTION BETWEEN INVESTMENT AND FINANCING DECISIONS: USE OF ADJUSTED NET PRESENT VALUE TECHNIQUE

Brealey and Myers suggest a methodology for analyzing the interactions between the investment decision, employing DCF and CAPM, and the financing decision. Their approach is to first compute the NPV of a project, assuming all-equity financing and using the DCF model with the discount rate determined using CAPM with asset betas. Under this base case, taxes are estimated assuming all-equity financing. The NPV is then adjusted to account for the effects of financing. The result is called adjusted NPV or APV. Brealey and Myers provide the following formula:

\[
APV = \text{base case NPV} + \text{NPV of financing decisions caused by acquisition of target}
\]

Brealey and Myers give two illustrations of financing effects. The first shows that if stock is to be issued to finance the project, the NPV of the issuance cost will cause APV to be less than the base case NPV. The second illustration shows that if part of the acquisition is to be financed by debt, the present value of the expected tax savings from the interest deductions will cause APV to exceed the base case NPV. They explain the process as follows:

A series of present value calculations is made . . . . The first establishes a base-case value for the project: its value as a separate, all-equity financed mini-firm. Then each side effect is traced out and the present value of its cost or benefit to the firm is calculated. Finally, all the present values are added together to estimate the project’s total contribution to the value of the firm. Thus, in general:

\[
\text{Project APV} = \text{base case NPV} + \text{sum of the present values of the side effects of accepting the project}
\]
Rappaport suggests that the function of the APV concept is similar to that of the weighted average cost of capital concept (WACC). WACC accounts for the financing decision by considering the cost of both debt and equity financing. Brealey and Myers point out three mistakes commonly made with WACC. First, the WACC formula works only for a project in the same line of business as the business normally carried on by the firm. Second, applying the concept to a discrete project may lead to the assumption that more debt should be used in financing the project, thereby lowering the discount rate and making the project look more attractive. On this point Brealey and Myers state:

[T]he immediate source of funds for a project has no necessary connection with the hurdle rate for the project. What matters is the project's overall contribution to the firm's borrowing power. A dollar invested in Q's pet project will not increase the firm’s debt capacity by 90 cents. If it borrows 90 percent of the project's cost, it is really borrowing in part against its existing assets. Any advantage from financing the new project with more debt than normal should be attributed to the old projects, not to the new one.

Third, using excessive debt in an acquisition will not necessarily reduce the cost of capital because the debt will increase the financial risk to shareholders and thereby increase the required rate of return on equity (i.e., the cost of equity).

As another alternative to WACC, Brealey and Myers suggest adjusting the cost of capital, determined using CAPM with asset betas, to reflect the "financing side effects of an investment project." Although they acknowledge that there is no generally accepted way of adjusting the discount rate, they discuss two useful rules of thumb. Their bottom line, however, is that in general the APV technique should be used. It appears that the proper application of either APV, the adjusted beta method, or WACC produces similar results because each measures the effects of the financing decision.

IX. USE OF THE WEIGHTED AVERAGE COST OF CAPITAL (WACC) IN DETERMINING THE APPROPRIATE DISCOUNT RATE

In certain situations it may be appropriate to discount a target's cash flows at the target's weighted average cost of capital (WACC). The WACC is determined by summing (1) the after-tax cost of debt, multiplied by the percentage of the value of the firm

370. See infra part IX (discussing WACC).
371. BREALEY & MYERS, supra note 6, at 469-70.
372. Id. at 470.
373. Id.
374. Id.
375. Id.; see also supra part VI.F (illustrating this common mistake in discussing adjustments to asset betas).
376. BREALEY & MYERS, supra note 6, at 459-60.
377. Id. at 461-64.
378. Id. at 475.
that is represented by debt, and (2) the expected return on equity, multiplied by the percentage of the value of the firm represented by equity.\textsuperscript{379}

The after-tax cost of debt is determined by multiplying the firm's current borrowing rate \( (r_d) \) by 1 minus the corporate tax rate \( (t) \), \((1 - t)\). This is the reciprocal of the tax rate. If the corporate tax rate is 34\%, \((1 - t)\) is equal to 66\% or .66. Thus, the after-tax cost of debt can be expressed as:

\[
\text{After tax cost of debt} = r_d(1 - t) = r_d(.66)
\]

The expected return on the firm’s equity \( (r_e) \) is a function of the firm’s business risk and financial risk, that is, the degree of leverage. The \( r_e \) can be computed by using CAPM and adjusting for financial risk.\textsuperscript{380} Thus, the equity beta can be used in the WACC model for determining the cost of equity. Alternatively, the cost of equity can be determined by using APT.\textsuperscript{381}

Shapiro suggests using the dividend growth model as a check on the cost of equity determined under CAPM or APT when determining the cost of equity capital for the WACC formula.\textsuperscript{382} Under the dividend growth model, the cost of equity is determined by:\textsuperscript{383}

\[
\text{Cost of Equity} = \frac{\text{Current Dividend}}{\text{Current Stock Price}} + \text{Expected Growth in Dividend}
\]

Thus, if the current dividend is $7, the current stock price is $100, the expected growth in the dividend is 5\%, and the cost of equity is 12\%:

\[
\text{Cost of Equity} = \frac{\$7}{\$100} + 5\% = 7\% + 5\% = 12\%
\]

The value of the firm \( (V) \) is equal to the sum of the value of the debt \( (D) \) and equity \( (E) \).

Thus, the WACC can be expressed algebraically as:

\[
\text{WACC} = r^d (1 - t) \frac{D}{V} + r^e \frac{E}{V}
\]
The following example illustrates the computation of WACC:

*Example (11).* Acquiring corporation (AC) has a cost of debt (rd) of 10%, implying that its after-tax cost of debt (rd)(1-.34) is therefore, 6.6%. AC’s cost of equity using CAPM and adjusting the beta for financial risk is 15%. Target corporation (TC) is in the same line of business as AC. AC plans to finance its acquisition of TC using AC’s historic debt to equity ratio—30% debt and 70% equity. Given these elements, AC’s WACC is computed as follows:

<table>
<thead>
<tr>
<th>Weight (%)</th>
<th>Cost of Capital (%)</th>
<th>Weighted Cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>30</td>
<td>6.6</td>
</tr>
<tr>
<td>Equity</td>
<td>70</td>
<td>15</td>
</tr>
<tr>
<td>Cost of Capital</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Professor Rappaport explains that WACC “incorporates the returns demanded by both debtholders and shareholders because *pre-interest* cash flows are discounted—that is, cash flow on which both debtholders and shareholders have claims.” He further states that the “appropriate cost of capital is, therefore, one that considers the claims of each group in proportion to its targeted relative capital contribution.”

WACC was one of the techniques utilized by Lazard in its valuation of Warner. In discounting Warner’s aggregate free cash flows from all of its business segments, Lazard used “a range about Warner’s average cost of capital” as discount rates. Brealey and Myers explain that the weighted average cost of capital is correctly used as a discount rate for the firm as a whole but not necessarily for a particular project:

The first thing to notice about the weighted-average formula is that all variables in it refer to the firm as a whole. As a result the formula gives the right discount rate only for projects that are just like the firm undertaking them. The formula works for the ‘average’ project. It is incorrect for projects that are safer or riskier than the average of the firm’s existing assets. It is incorrect for projects whose acceptance would lead to an increase or decrease in the firm’s debt ratio.

---

384. See, e.g., RAPPAPORT, supra note 6, at 55.
385. Id.
386. Id.
387. See infra Appendix D, at D-4.
388. See id., at D-4.
389. BREALEY & MYERS, supra note 6, at 465.
Brealey and Myers suggest that the projected free cash flows of each segment should be discounted at the appropriate discount rate for that segment determined by using CAPM. Conversely, Professor Rappaport supports use of the “weighted average cost of debt and equity capital” as the “appropriate rate for discounting a company’s cash flow stream.” Professor Rappaport suggests that, in discounting the cash flows of a potential target, the discount rate determined under CAPM should be used only for acquisitions financed by 100% equity. In that case the weighted average cost of capital equals the cost of equity determined under CAPM. Whenever debt is used in the acquisition, the discount rate using the weighted average cost of capital is less than that obtained using CAPM because the cost of equity exceeds the cost of debt. Also, greater debt and a lower weighted average cost of capital results in a higher price for the target.

The basic difference between the WACC approach suggested by Professor Rappaport and the all-equity (or asset beta) approach suggested by Brealey and Myers is that WACC takes account of the financing decision in the application of the DCF model, whereas Brealey and Myers suggest analyzing any debt financing benefits separately.

X. APPLICATION OF DCF IN ACQUISITIONS THAT PRODUCE SYNERGIES

In applying the DCF and NPV techniques, any synergistic (or merger) gains resulting from the acquisition must be taken into account. Synergistic gains increase cash flow as a result of either an increase in revenues or a decrease in cost. Professor Rappaport has stated that synergistic gains fall into three principal categories: operating, financial, and tax.

Brealey and Myers explain that in preparing a merger analysis, some firms include synergistic gains in the forecast of the target’s cash flows and then discount the cash flows to present value and compare the present value with the purchase price. They describe this process as:

\[
\text{Estimated Net gain} = \frac{\text{DCF valuation of target, including merger benefits}}{-\text{Cash required for acquisition}}
\]

The “estimated net gain” is the same as the NPV resulting from the merger. This process, referred to here as the standard approach, is illustrated as follows:

Example (12). Assume that an acquiring firm has a preacquisition trading value of $200 million and a target has a preacquisition trading value of $60 million. A DCF analysis of the target’s cash flows indicates, however, that

390. Id. at 181.
391. RAPPAPORT, supra note 6, at 55.
392. See supra part VII (discussing the Brealey and Myers approach).
393. Cf. WESTON ET AL., MERGERS, supra note 2, at 194.
394. RAPPAPORT, supra note 6, at 207-15.
395. BREALEY & MYERS, supra note 6, at 820.
the target has a present value of $100 million. The acquisition requires $90 million in cash. Therefore, the estimated net gain, or NPV, resulting from the acquisition is $10 million. After the acquisition, assuming the transaction is financed with $90 million from the sale of new equity, the combined firm will consequently have an expected value of $300 million.396

Brealey and Myers have stated that the standard NPV process "is a dangerous procedure. Even the brightest and best-trained analyst can make large errors in valuing a business. The estimated net gain may come up positive not because the merger makes sense, but simply because the analyst's cash flow forecasts are too optimistic."397 As an alternative to this standard NPV approach, Brealey and Myers suggest a merger analysis that "starts with the target's stand-alone market value . . . and concentrates on changes in cash flow that would result from the merger."398 Professor Rappaport suggests a similar approach.399 As demonstrated infra, this alternative approach produces the same result as the standard NPV approach, provided the two approaches are properly applied. Further, this alternative approach is subject to the same flaw as the standard NPV approach: the possibility of an inaccurate DCF analysis.

The alternative approach starts by determining if there is an economic gain resulting from the acquisition.400 Brealey and Myers explain that there is an "economic gain only if the two firms are worth more together than apart."401 This concept can be expressed as:

\[
\text{Gain Resulting from Acquisition} = \frac{\text{Value of Combined Firms}}{\text{Preacquisition Value of Acquiror}} + \text{Value of Target}
\]

Under this formula, the gain resulting from the acquisition in Example (12) is $40 million, the estimated value of the combined firms ($300 million) minus the sum of the preacquisition value of both the acquiror ($200 million) and the target ($60 million).

Since the acquisition results in a gain, there is an "economic justification" for the acquisition.402 This does not mean, however, that the acquiror should make the acquisition. This question depends upon the division of the gains between the target's shareholders and the acquiror's shareholders. There is a positive NPV to the acquiror's shareholders only if the cost of the acquisition is not more than the gain resulting from the

396. The expected value represents the $200 million preacquisition trading value of the acquiror $90 million purchase price of target $10 million NPV.
397. BREALEY & MYERS, supra note 6, at 820.
398. Id.
399. RAPPAPORT, supra note 6, at 202-07.
400. BREALEY & MYERS, supra note 6, at 817.
401. Id. at 817-18.
402. Id. at 818; see also RAPPAPORT, supra note 6, at 202.
403. BREALEY & MYERS, supra note 6, at 818.
acquisition. In a cash acquisition, the cost of the acquisition is the difference between the cash paid to the target’s shareholders and the preacquisition value of the target:

\[
\text{Cost} = (\text{Cash}) - (\text{Preacquisition Value of Target})
\]

In the above example, this cost is $30 million, the difference between the $90 million purchase price and the $60 million preacquisition trading price of the target.

The NPV to the acquiror’s shareholders is the difference between acquisition gains and cost:

\[
\text{NPV} = \text{gain} - \text{cost} = \begin{bmatrix}
\text{Value of Combined Firms} \\
($300\text{m})
\end{bmatrix} - \begin{bmatrix}
\text{Preacquisition Value of Acquiror} \\
($200\text{m})
\end{bmatrix} + \begin{bmatrix}
\text{Preacquisition Value of Target} \\
($60\text{m})
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Cash} \\
($90\text{m})
\end{bmatrix} - \begin{bmatrix}
\text{Preacquisition Value of Target} \\
($60\text{m})
\end{bmatrix}
\]

Based on the facts in Example (12), this formula produces a NPV of $10 million, the difference between the $40 million gain and the $30 million cost. This NPV equals the NPV calculated using the standard method shown in Example (12):

\[
\text{NPV} = (\text{PV of Target ($100,000,000)}) - (\text{Cost ($90,000,000)})
\]

As long as all the incremental cash flows, including those from synergistic gains, are properly identified,\(^{405}\) the two approaches should produce the same result. Finally, if the acquisition of the target is financed with stock of the acquiror, the target’s shareholders will automatically share in any NPV benefits generated by the merger through their ownership of the acquiror’s stock. Therefore, this factor should be taken into account in determining the amount of acquiror’s stock to be transferred to the target’s shareholders.\(^{406}\)

Further, it appears that the alternative approach functions only when the target is acquired either for stock of the acquiror or for cash raised by a new stock issuance of the acquiror. Only those cases appropriately consider the value of the stock of the combined firms as including the preacquisition value of the acquiror and target, which is implicit in the alternative approach.\(^{407}\) For example, if the acquisition is completely debt-financed, the equity value of the combined firm will equal the equity value of the

\[\text{Id.}^{404}\]

\[\text{See supra part V.D (discussing identification of incremental cash flows).}^{405}\]

\[\text{See generally BREALEY & MYERS, supra note 6, at 830; SHAPIRO, supra note 6, at 936-37.}^{406}\]

\[\text{The alternative approach could possibly be used in a debt-financed acquisition if the value of the two firms is considered to equal both the debt and equity of the firms.}^{407}\]
acquiror plus the NPV of the transaction. The NPV, therefore, is the value added for the acquiror’s shareholders.

Since both approaches are subject to the same potential flaws, and the standard approach is used in all circumstances, the standard approach should be the preferred technique. The alternative can be used as a reality check in appropriate cases.

XI. ILLUSTRATION OF THE APPLICATION OF DCF, NPV, AND CAPM IN AN ACQUISITION CONTEXT

Acquiring corporation (AC) is analyzing the proposed acquisition of target corporation (TC) using DCF, NPV, and CAPM. The asking price of TC is $6 million. AC follows the five steps set forth in Part III. First, AC estimates the amount and timing of the free cash flows and the terminal value of TC.

<table>
<thead>
<tr>
<th>Estimated Free Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The terminal value of TC at the end of the fifth year is estimated to be $7 million. Thus, the cash flows are:

<table>
<thead>
<tr>
<th>Free Cash Flow</th>
<th>Terminal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$600,000</td>
</tr>
<tr>
<td>2</td>
<td>$700,000</td>
</tr>
<tr>
<td>3</td>
<td>$800,000</td>
</tr>
<tr>
<td>4</td>
<td>$900,000</td>
</tr>
<tr>
<td>5</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>$7,000,000</td>
</tr>
</tbody>
</table>

There are two expected cash flows in year five because TC will likely throw off $1 million in operating cash. The balance of its assets will be sold for the terminal value of $7 million.

Second, AC determines the appropriate discount rate \( r \) by utilizing CAPM. Assume that AC is buying all the stock of TC, which is debt free, and the acquisition will be financed with existing cash. If the target has outstanding debt to be assumed, the following methodology should still be utilized. However, after determining the value of

409. See infra Appendix C, at C-2; supra part V.H (setting forth the cash flow statement in Example 5, providing an estimate of the free cash flow for the five-year period AC intends to operate TC).
410. See supra part V.I.3 (computing the $7 million terminal value in Example (8) using the competitive equilibrium formula).
411. See infra Appendix C, at C-3. This chart does not indicate how the discount rate was determined.
the target, the value of the assumed debt is subtracted to determine the value of the equity. Also, the value of any marketable securities or other investments not accounted for in computing free cash flows and terminal value are added when computing the value of TC.

Assume that TC operates one line of business. AC first determines the asset beta for TC’s line of business, probably through the use of a beta book. If only an equity beta is available, the beta will have to be adjusted (i.e., unleveraged) to arrive at the asset beta. If there are multiple lines of business, separate asset betas and separate discount rates would be determined for each business.

Assume that the asset beta is 1.2. TC then determines the risk-free ($r_f$) rate, assumed to be 5%. AC takes the market risk premium ($r_m - r_f$) as the 8.4% historic premium. These elements are plugged into the CAPM model as follows:

$$ r = r_f + \beta(r_m - r_f) $$

$$ r = 5\% + 1.2(8.4\%) $$

$$ r = 5\% + 10.08\% $$

$$ r = 15.08\% $$

Thus, the appropriate discount rate is 15.08% (rounded to 15%). This rate does not have to be adjusted because the acquisition is financed with available cash.

If the acquisition was partially financed with debt, then at least three adjustment methods could be used. First, if TC was in the same line of business as AC and the acquisition was financed with the same proportions of debt and equity as AC had outstanding, then AC could use its weighted average cost of capital (WACC) as the discount rate. Second, as Brealey and Myers proposed, the discount rate determined using the asset beta could be adjusted to reflect the debt financing. Either method will produce a discount rate lower than the rate determined using the asset beta. Third, the adjusted net present value (APV) method suggested by Brealey and Myers could be utilized to convert the expected benefits from the interest deduction on the debt to present value. Each method would increase the net present value of the project over the net present value produced with the asset beta. If each method is properly employed, the bottom line results (i.e., NPV for the project) should be the same.

---

412. See COPELAND ET AL., VALUATION, supra note 6, at 97-103; see also infra Appendix C, at C-3.
413. See infra Appendix C, at C-7.
414. See SHAPIRO, supra note 6, at 279-81 (discussing the unleveraging process).
415. See supra note 306 and accompanying text.
416. See supra part IX (discussing WACC).
417. See supra part VIII (discussing the interaction between the financing decision and the investment decision).
418. Id.
The third step in the process requires that the free cash flows and terminal values are discounted to present value at the applicable discount rate using the DCF model:

\[ PV = \frac{1}{(1 + r)^1} F_{A_1} + \frac{1}{(1 + r)^2} F_{A_2} + \frac{1}{(1 + r)^3} F_{A_3} + \frac{1}{(1 + r)^4} F_{A_4} + \frac{1}{(1 + r)^4} TV \]

\[ PV = \frac{600,000}{1 + .15} + \frac{700,000}{(1 + .15)^2} + \frac{800,000}{(1 + .15)^3} + \frac{900,000}{(1 + .15)^4} + \frac{1,000,000}{(1 + .15)^4} + \frac{7,000,000}{(1 + .15)^4} \]

\[ PV = .870(600,000) + .756(700,000) + .658(800,000) + .572(900,000) + .497(1,000,000) + .497(7,000,000) \]

\[ PV = 522,000 + 529,200 + 526,400 + 514,800 + 497,000 + 3,479,000 \]

During the fourth step, these amounts must be added to determine the present value of all the cash flows and the terminal value: $6,068,400. This amount can be derived using a business calculator by entering the cash flows and terminal values for each period and then discounting these amounts at the 15% rate.

Finally, AC computes the net present value (NPV) of the investment by subtracting the initial cost of TC (the asking price of $6 million), from the present value of the cash flows and terminal value ($6,068,200), determining that the proposed investment has a net present value of $68,200. Thus, this project could be undertaken by AC. Of course, AC would want to offer something less than $6 million possibly increasing the project’s NPV.

If AC has other potential projects with positive NPVs, AC should select the project or projects indicated by AC’s capital rationing model, such as the profitability index. Before proceeding, however, AC may want to do a sensitivity analysis, particularly since the project only has a NPV of $68,200, assuming a $6 million purchase price.

XII. NON-DCF VALUATION TECHNIQUES: COMPARABLE COMPANIES AND COMPARABLE TRANSACTIONS

A. Introduction to the Direct Comparisons

Professor Cornell states that “[e]conomic theory and common sense agree on the basic principle that similar assets should sell at similar prices.” Consequently, the value of an asset (i.e., a target) may be determined by referring to the value of a com-
parable asset (i.e., a comparable company) recently sold by a reasonably informed seller to a reasonably informed purchaser. Professor Cornell refers to this as the direct comparison approach.424

If the size of the comparable company is not identical to the size of the target, which will normally be the case, the size difference can be adjusted for by identifying an observable variable related to the price or value of both the comparable company and the target. The fundamental assumption of the direct comparison approach is that the ratio of the price \( P \) of the comparable company to the observable variable \( X \) of the comparable company is approximately equal to the ratio of the value \( V \) of the target to the observable variable \( X \) of the target.425 This approach works if both the price and observable variable are available for the comparable company and the observable variable is available for the target.426

Where \( V_T \) is the value of the target, \( X_T \) is the observable variable of the target, \( P_C \) is the price of the comparable firm or firms, and \( X_C \) is the observable variable of the comparable firm or firms, the above principle may be expressed as:427

\[
\frac{V_T}{X_T} = \frac{P_C}{X_C}
\]

The only unknown variable is \( V_T \), which can be determined as:428

\[
V_T = X_T \cdot \frac{P_C}{X_C}
\]

For example, assume that (1) the trading price of the comparable company \( P_C \) is $100 per share, (2) the observable variable \( X_C \) is earnings per share of $20, and (3) the observable variable of the target \( X_T \) is earnings per share of $10. The per share value of the target \( V_T \) is fifty dollars:

\[
\begin{align*}
V_T &= $10 \times \frac{P_C}{X_C} \\
V_T &= $10 \times \frac{100}{20} \\
V_T &= $50
\end{align*}
\]

The above formula merely derives the per share value of the target by multiplying the target’s earnings per share by the price earnings ratio of the comparable firm.

Professor Cornell points out that “a critical step in applying the direct comparison approach is choosing observable variables, \( X \), that have a consistent relationship to val-

424. Id.
425. Id. at 57.
426. Id.
427. CORNELL, CORPORATE VALUATION, supra note 6, at 57.
428. Id.
He further explains that in valuing companies, observable variables such as "cash flow and earnings are good choices because the ultimate sources of value are the net benefits received by investors." He

B. Equivalence Between Direct Comparison and Direct Capitalization

In the direct comparison approach, in determining the value of the target ($V_T$), the observable variable of the target ($X_T$) is multiplied by the ratio of the price of the comparable firm or firms ($P_C$) to the observable variable of the comparable firm or firms ($X_C$):

$$V_T = X_T \cdot \frac{P_C}{X_C}$$

An equivalent alternative approach to valuing the target is the direct capitalization method, in which the value of the target ($V_T$) is the observable variable of the target ($X_T$) divided by the observable variable of the comparable firm or firms ($X_C$) to the price of the comparable firm or firms ($P_C$):

$$V_T = \frac{X_T}{X_C/P_C}$$

The denominator of the above equation ($X_C/P_C$) is known as the capitalization rate, the rate at which the observable variable of the target is capitalized to ascertain the value of the target.

The equivalence of the direct comparison and direct capitalization methods can be demonstrated under the assumption that $X_T = \text{ten dollars per share}$, $P_C = \$100 \text{ per share}$ and $X_C = \text{twenty dollars per share}$:

<table>
<thead>
<tr>
<th></th>
<th>DIRECT COMPARISON</th>
<th>DIRECT CAPITALIZATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_T$</td>
<td>$X_T \cdot \frac{P_C}{X_C}$</td>
<td>$V_T = \frac{X_T}{X_C/P_C}$</td>
</tr>
<tr>
<td>$V_T = $10 \cdot \frac{$100}{$20}$</td>
<td>$V_T = \frac{$10}{$20/$100}$</td>
<td></td>
</tr>
<tr>
<td>$V_T = $10 \cdot 5$</td>
<td>$V_T = \frac{$10}{.2}$</td>
<td></td>
</tr>
<tr>
<td>$V_T = $50$</td>
<td>$V_T = $50$</td>
<td></td>
</tr>
</tbody>
</table>

429. _Id._

430. _Id.; see also infra_ Appendix C, at C-5; Appendix D, at D-6 (demonstrating that both Lazard and Wasserstein/Shearson used the direct comparison approach in valuing Warner).

431. _CORNEILL, CORPORATE VALUATION, supra_ note 6, at 57-59.
The above equations demonstrate that in determining the target’s value under the direct comparison approach, the target’s observable variable (earnings per share) is multiplied by the price to earnings ratio \( \left( \frac{P}{E} \right) \) of the comparable company. In determining the target’s value under the direct capitalization approach, the target’s earnings per share is divided by the earnings to price ratio \( \left( \frac{X}{P} \right) \) of the comparable company.

Professor Cornell explains that the capitalization rate should not be confused with the cost of capital used in the DCF model. He states that “situations in which the cost of capital can be used as a capitalization rate are extraordinarily rare.”

C. Selecting Comparable Companies and Determining Multiples

The selection of comparable companies invariably involves judgment. One approach focuses on companies doing business in the same standard industrial classification (SIC) code, published in the federal government’s Standard Industrial Classification Manual. Another approach focuses on reports of securities analysts, which often contain lists of comparable firms. Yet another approach is to review reports of investment research firms such as Standard & Poor’s, Moody’s, and ValueLine. Industry experts, including operations personnel at both the target and the acquiror, may also assist in identifying comparable companies.

Once the comparable companies are selected, the relationship between the companies’ trading price and the observable variable such as the earnings are analyzed. For example, if the price to earnings (P:E) ratio is used, a mean and median P:E ratio for the companies may be computed. In the P:E ratio the value indicator \( P \) is the comparable firms’ equity trading price, and the observable variable \( E \) represents the after-tax earnings of such firms. If a company has a significantly higher or lower P:E ratio than is representative of the group, such “outliers” may be eliminated.

The following example illustrates the above process:

<table>
<thead>
<tr>
<th>Comparable Company</th>
<th>Equity Value/1995 Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

Average 100/5 = 20
Median = 20

432. See supra parts VI and VII (discussing CAPM and APT, respectively).
433. CORNELL, CORPORATE VALUATION, supra note 6, at 58-59.
434. Id. at 60-69.
435. Id. at 68.
436. See, e.g., id. at 67.
If the target's 1995 earnings are $10 million, then the implied value of the target's equity is $200 million—twenty times $10 million.

In many cases the P:E ratio is computed to determine the estimated earnings of the comparable companies for the next year. Determining the target value requires applying the average and median ratios to the target's estimated earning for the next year.\textsuperscript{437} Professor Cornell states:

The problem with actual earnings is that they reflect year-to-year fluctuations in profitability that may not be permanent and, therefore, may have little [impact] on value. Because analysis cannot anticipate such random year-to-year fluctuations, estimated earnings are based more on assessments of long term trends. As a result, estimated earnings tend to vary less over time and from company to company.\textsuperscript{438}

D. Selecting the Value Indicator and the Observable Variable\textsuperscript{439}

In the P:E ratio, the value of the common equity is the value indicator (\(P\)). The earnings, net of interest, taxes, and preferred dividends is the observable variable (\(E\)). Although the P:E ratio is commonly used in valuing common equity, this method faces major potential defects.

First, the capital structure of firms can vary significantly, which can significantly impact earnings. If Firm A, for example, is financed with 50% debt and Firm B has no debt, then, other things being equal, Firm B will have higher earnings because it has no interest deduction:

<table>
<thead>
<tr>
<th></th>
<th>Firm A</th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$200K</td>
<td>$200K</td>
</tr>
<tr>
<td>Operating Expense</td>
<td>-40K</td>
<td>-40K</td>
</tr>
<tr>
<td>Interest</td>
<td>-50K</td>
<td>0</td>
</tr>
<tr>
<td>Income Before Taxes</td>
<td>110K</td>
<td>160K</td>
</tr>
<tr>
<td>Taxes (50%)</td>
<td>-55K</td>
<td>-80K</td>
</tr>
<tr>
<td>After Tax Earnings</td>
<td>$ 55K</td>
<td>$ 80K</td>
</tr>
</tbody>
</table>

Thus, use of the P:E ratio may produce distorted results if the capital structure of the comparable firms differ from the capital structure of the target. Second, the accounting practices used by the comparable firms may differ from those used by the target. For example, firms may use different depreciation or amortization methods. Also, earnings may reflect extraordinary items.

An alternative to the use of the P:E ratio is the use of total firm value (TFV)—the value of the firm's debt and equity—as the value indicator, and use of a broad measure of earnings, such as earnings before interest, taxes, depreciation, and amortization.
(EBITDA) as the observable variable \( E \). Professor Cornell has examined the theoretical reason for broadly measuring both firm value and earnings:

The view that broader measures of income are more appropriate is supported by the work of Miller and Modigliani. In several classic research papers, Modigliani and Miller present convincing arguments showing that the value of the firm depends little, if at all, on the firm’s capital structure. Therefore, the ratio of total firm value to a gross measure of income such as [EBITDA] should be similar even for firms with different capital structures.\(^4\)

The advantage of using the ratio of TFV to EBITDA can be illustrated. Assume that Firms A and B are identified as comparable firms to the target. Firm A is financed with 50% debt, with a principal amount of $500,000. The interest rate on the debt is 10%, or $50,000 per year. The trading value of Firm A’s common stock is $1 million. Firm B has no debt but has an amortization deduction attributable to an acquisition it made. This amortization deduction is not accounted for in computing Firm B’s taxes. The trading value of Firm B’s common stock is $1.5 million. The reported earnings of Firm A and Firm B are:

<table>
<thead>
<tr>
<th></th>
<th>Firm A</th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$200K</td>
<td>$200K</td>
</tr>
<tr>
<td>Operating Expense</td>
<td></td>
<td></td>
</tr>
<tr>
<td>other than depreciation</td>
<td>-30K</td>
<td>-30K</td>
</tr>
<tr>
<td>Depreciation</td>
<td>-20K</td>
<td>-20K</td>
</tr>
<tr>
<td>Interest</td>
<td>-50K</td>
<td>-0</td>
</tr>
<tr>
<td>Income Before Taxes</td>
<td>100K</td>
<td>150K</td>
</tr>
<tr>
<td>Taxes (50%)</td>
<td>-50K</td>
<td>-75K</td>
</tr>
<tr>
<td>After Tax Earnings</td>
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<td>$ 75K</td>
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<tr>
<td>Amortization Deduction</td>
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<td>-10K</td>
</tr>
<tr>
<td>Reported Earnings</td>
<td>$ 50K</td>
<td>$ 65K</td>
</tr>
</tbody>
</table>

The P:E ratio of Firm A is twenty to one:

\[
\text{Price of Common Stock$1,000,000 : Reported Earnings$50,000} \\
= \frac{1,000,000}{50,000} \\
= 20:1
\]

The P:E ratio of Firm B is twenty-three to one:

\[
\text{Price of Common Stock$1,500,000 : Reported Earnings$65,000} \\
= \frac{1,500,000}{65,000} \\
= 23.1:1
\]

\(^4\) Id. at 71.
The EBITDA of Firms A and B are:

<table>
<thead>
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<th></th>
<th>Firm A</th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$200,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>Operating Expense</td>
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</tr>
<tr>
<td>EBITDA</td>
<td>$180,000</td>
<td>$180,000</td>
</tr>
</tbody>
</table>

Thus, when this broader measure of earnings is used, the firms have comparable earnings.

Here, the ratio of total firm value (TFV) can be used as the value indicator, and EBITDA can be used as the observable variable. Since Firm A has debt valued at $500,000 and common equity valued at $1 million, the TFV of Firm A is $1.5 million. Firm B has common stock with a value of $1.5 million and no debt. Therefore, its TFV is also $1.5 million. Since both Firm A and Firm B have the same TFV's and EBITDA, they each have an 8.3 to one ratio of TFV to EBITDA:

\[
\text{TFV} \left( \frac{\$1,500,000}{\$180,000} \right) = 8.3:1
\]

If the target has EBITDA of $100,000, the estimated TFV of the target would be $830,000:

\[
\text{(Target's EBITDA)} \times \left( \frac{\text{TFV:EBITDA Ratio of Comparable Firms}}{8.3} \right) = \text{Estimated TFV}
\]

\[
\text{($100,000) \times (8.3) = $830,000}
\]

The value of the target's common stock is $830,000, minus the value of its debt and preferred stock.

In valuing a firm's debt and preferred stock for purposes of determining the total firm value (TFV), Professor Cornell posits that only comparable securities with comparable risk must be found. It is not necessary to find comparable firms. Professor Cornell elaborates:

It may seem paradoxical that firms that are not comparable can issue comparable securities, but the paradox has a simple explanation. As long as a company is not so highly leveraged that there is a significant risk of default on its fixed income securities, the prices of those securities depend primarily on the size of the promised payments and on the level of interest rates. Therefore, long-term debt or preferred stock issued by a medical supply company will be comparable to long-term debt or preferred stock issued by a computer company as long as the risk of default is similar.\footnote{Id. at 73.}
In certain cases it may be appropriate to adjust the observable variable, such as earnings or EBITDA, to account for abnormalities.\footnote{442} For example, statistical methods may be used to calculate five-year averages in earnings. This average would reduce the impact of abnormalities in earnings. As an alternative to statistical adjustments, the earnings or other observable variable may be directly adjusted to account for specific abnormalities.\footnote{443}

E. Accounting for Control Premiums: The Comparable Transactions Approach

In identifying the value indicator, the current trading price of common equity of comparable firms may be used. However, this may understate the value of the target because the current trading price may not reflect a "control premium".\footnote{444} The price reflecting a control premium is probably a better value indicator in determining the value of the target in a change of control transaction. Identifying comparable companies that have undergone comparable transactions is important in focusing on a control premium. Alternatively, a control premium may be added to the value determined using the trading value as the value indicator. Professor Cornell indicates that "it is safe to say that the estimate of value derived from the market prices of securities establishes a floor beneath a company's value. Under certain circumstances, the appraiser may choose to elevate the appraised value above that floor to take into account a control premium."\footnote{445}

F. Applicability of the Direct Comparison Approach in Time-Warner

Both Wasserstein/Shearson\footnote{446} and Lazard\footnote{447} used the direct comparison approach in valuing Warner, and both used the comparable transaction approach and the comparable companies approach. One of the Value Indicators Wasserstein/Shearson used in its comparable transactions model was transaction value Net Debt, comprising the total firm value (TFV). Both Net Sales and EBIT were used as observable variables, constituting two broad measures of economic activity. A mean, median, high, and low were computed in determining the multiple of Net Sales (EBIT) to TFV (the ratio between TFV and Net Sales, or TFV and EBIT). The implied value of the assets of the Film Entertainment segment was then determined by multiplying the 1989 Net Sales and EBIT for that segment by the mean, median, high, and low multipliers.\footnote{448}

A similar procedure was followed with Wasserstein/Shearson's comparable companies analysis,\footnote{449} except that a 50% control premium was added to the implied asset

\begin{footnotes}
\footnotetext[442]{Id. at 75-87.}
\footnotetext[443]{CORNELL, CORPORATE VALUATION, supra note 6, at 83-86.}
\footnotetext[444]{Id. at 87.}
\footnotetext[445]{Id.; see also Kleinwort Benson LTD. v. Silgon Corp., 1995 WL 376911, at *2-4 (Del. Ch. 1995) (discussing control premiums).}
\footnotetext[446]{See infra Appendix C, at C-4 (illustrating Wasserstein/Shearson's use of the comparable transaction approach); Appendix C, at C-5 (illustrating the comparable companies analysis).}
\footnotetext[447]{See infra Appendix D, at D-5 (illustrating Lazard's use of the comparable companies approach); Appendix D, at D-6 (illustrating the comparable transaction analysis).}
\footnotetext[448]{See infra Appendix C, at C-4 (illustrating Wasserstein/Shearson's comparable transactions approach).}
\footnotetext[449]{See infra Appendix C, at C-5 (illustrating the comparable companies approach).}
\end{footnotes}
values. The addition of a control premium to the trading value is one of the techniques suggested by Professor Cornell.450

Lazard uses two value indicators in its comparable companies approach:451 (1) market cap—the total value of the firm's equity and debt, and (2) market value—the value of the firm's common stock. Lazard used: (1) revenues, (2) EBITDA, (3) operating income (i.e., income after depreciation), and (4) net income (i.e., income after depreciation, interest, and taxes) as observable variables, ranging from the broadest (revenues) to the narrowest (net income). For net income, the ratio is market value (i.e., value of common equity) to net income. The ratio for the other three observable variables is market cap (i.e., TFV) to revenues, market cap to EBIT, and market cap to operating income, respectively. The low and high multiples for each of these ratios are computed for the selected comparable public companies and these multipliers are used in computing the implied value range of Warner based on its 1989 estimated revenues, EBITDA, operating income, and net income.

Lazard used a similar process in analyzing comparable acquisitions.452 Only the market cap and market value used therein differ, in that they reflect a control premium.

XIII. CONCLUSION

DCF, CAPM, APT, and WACC are powerful economic models used in making capital budgeting decisions, including the valuation of a target firm in a merger or acquisition context. Although the concepts behind these models may appear mystical, the application of the concepts is straightforward in the acquisition context. A five-step process for applying these concepts is utilized:

First, the periodic and terminal free and negative cash flows that the target is expected to produce is estimated. In making these projections, all-equity financing is assumed so that there is no deduction for interest in computing taxes. The expected actual tax liability, accounting for depreciation, is also deducted. Depreciation and other non-cash expenses are added back, and expected costs of future capital improvements are deducted. Thus, this projects the target's pure cash flows.453 If CAPM or APT is used to determine the discount rate, the cash flows associated with costs (such as stock or debt issuance costs) and benefits (such as tax benefits from the interest deduction) of the financing structure are separately projected.454 This is not done if WACC is used.

Second, the appropriate discount rate for the target utilizing CAPM, APT, or WACC is determined.455 If the target has several lines of business, the appropriate discount rate must be determined for each business. The use of WACC is appropriate only if the target is engaged in the same line of business as the acquirer, and the acquisition is financed with the same proportion of debt and equity that the acquirer has outstand-

450. See supra part XII.E (discussing the treatment of control premiums).
451. See infra Appendix D, at D-5 (illustrating Lazard's comparable companies approach at part III).
452. See infra Appendix D, at D-6 (illustrating Lazard's comparable transaction approach at part IV).
453. See supra part V (discussing the estimation of free cash flows).
454. See supra part VIII (discussing the interaction between the financing decision and the investment decision).
455. See supra parts VI, VII, IX (discussing CAPM, APT, and WACC, respectively).
ing. If WACC is used, the cost of equity will generally be determined through the use of CAPM or APT.  

Third, using the DCF model and the appropriate discount rate, the target's expected periodic and terminal free and negative cash flows are discounted to present value. If CAPM or APT is used, the expected cash flows from the costs and benefits of the financing structure, such as the tax savings from the interest deduction, will be discounted with the other cash flows from operations. If WACC is used, the tax benefit of the interest deduction is automatically taken into account in determining the cost of debt, leading to a cost of capital lower than would be the case using CAPM or APT.  

Fourth, the present value of the target's cash flows is determined by adding the positive present values of the free cash flows and the negative present values of the negative cash flows. Any outstanding debt that the target has is subtracted and the value of any investment securities and other such items is added in computing the present value.

Fifth, the net present value (NPV) is determined by subtracting the purchase price for the target from the present value of the target. If CAPM or APT is used in determining the cost of capital, and the costs and benefits of the financing decision are discounted, the NPV is referred to as adjusted NPV (APV), because the present value of the financing costs and benefits are taken into account. If the cost of capital can be appropriately determined by either using WACC, on the one hand, or CAPM or APT, on the other, the final NPV will be the same provided the financing effects are properly accounted for in applying the particular method.

Finally, in applying the valuation steps, Brealey and Myers have warned that: "even the brightest and best-trained analyst can make large errors in valuing a business. The estimated net gain [i.e., NPV] may come up positive not because the merger makes sense, but simply because the analyst's cash flow forecasts are too optimistic." To this warning should be added another: a positive NPV may also be attributable to an inappropriately low discount rate. Thus, whenever there is a positive NPV, the valuation analyst should clearly demonstrate that the acquisition is likely to produce economic rents, that is, excess profits.

456. *See supra* part IX (discussing WACC).
457. *Id.*
458. *Id.*
459. BREALEY & MYERS, *supra* note 6, at 820.
APPENDIX A

EXCERPTS FROM JOINT TIME AND WARNER INFORMATION STATEMENT AND TIME PROSPECTUS DATED DECEMBER 6, 1989

The Company. [Warner Communications] At the June 16, 1989 meeting of the Company’s Board of Directors all of the directors present at such meeting unanimously approved and adopted the Merger Agreement and recommended that all stockholders accept the Offer and tender their shares in the Offer.

* * * *

In making the determination on June 16, 1989 that the Offer was, and the Merger is, fair to, and in the best interests of, the stockholders of the Company and recommending that stockholders of the Company tender their shares in the Offer and approve and adopt the Merger Agreement, the Board of Directors of the Company considered a number of factors, including the following:

* * * *

(iv) The oral and written presentations of Lazard delivered to the Board of Directors at its meeting on June 16, 1989, including Lazard’s oral opinion to the effect that, as of June 16, 1989, the consideration to be received in the Offer and the Merger, when taken together, is fair to the stockholders of the Company from a financial point of view (a copy of such opinion as confirmed in writing and setting forth the assumptions made, matters considered, and procedures followed by Lazard, is discussed herein under “—Opinions of Financial Advisors” and set forth in Annex F hereto);

* * * *

(d) The oral presentation of Lazard referred to in paragraph (iv) above included a detailed summary by representatives of Lazard of its written presentation, which is described under the heading “Fairness Opinion of the Company’s Financial Advisor” and which is included as an exhibit to the Schedule 13E-3. The Lazard Presentation (as defined below) set forth ranges of values by a variety of valuation analyses of the Company, including pre-tax segment valuations of the Company and its businesses, as well as discounted cash flow and equity present value analyses. See “Opinions of Financial Advisors—Fairness Opinion of the Company’s Financial Advisors”. In connection with their consideration of such information, the directors asked the Lazard representatives questions concerning the presentation and commented on the content of the presentation. This presentation
was strongly favorable to the Board’s determination and while the Board did not take any specific action with respect to the presentation, in making its favorable determination regarding the Merger, the Board accepted the overall analyses and conclusions of the Lazard Presentation.

* * *

Fairness Opinion of the Company’s Financial Advisor

Lazard has delivered its written opinion to the Board of Directors of the Company to the effect that, as of June 16, 1989, the consideration to be received by the Company’s stockholders in the Offer and the Merger, when taken together, is fair to the Company’s stockholders from a financial point of view. A copy of this opinion of Lazard is attached hereto as Annex F 1. Company stockholders are urged to read this opinion in its entirety for assumptions made, matters considered and limits of the review by Lazard.

* * *

The information contained in the Lazard Presentation, including the pre-tax segment valuation of the Company and its businesses and the discounted cash flow and equity present value analyses, was based on the information reviewed and factors considered by Lazard in rendering its fairness opinion, discussed above, and does not reflect, among other things, actual financial results of the Company to date, revised prospects for the Company’s businesses, changes in general business and economic conditions or any other transaction or event that has occurred or that may occur and that was not anticipated at the time such information was prepared, including the recent sale of a major motion picture studio.

(i) Pre-tax Segment Valuation Analyses. Lazard performed its pre-tax segment valuation of the Company and its businesses through discounted cash flow analyses, comparable acquisitions analyses and analyses of the public market trading values and private market values of comparable companies for each of the filmed entertainment, recorded music/music publishing, cable and broadcasting and publishing and related distribution segments of the Company. Such analyses were based in part on certain projections provided to Lazard by the Company. See “—Certain Estimated Financial Information”. Based on such projections and analyses, and subject to the assumptions set forth in the Lazard Presentation, Lazard estimated that, based on the various analyses described above, the Company had a pre-tax hypothetical break-up value at the time of the Lazard Presentation of between $13.05 billion and $15.36 billion or between $65.92 and $77.57 per share.

1. See Appendix B.
2. See Appendix D, pages D-5 and D-6 (regarding the film segment).
(ii) *Discounted Cash Flow and Equity Present Value Analyses.* The Lazard Presentation also contained a consolidated unleveraged discounted cash flow analysis of the Company, which was based in part on the projections provided to Lazard by the Company. See "—Certain Estimated Financial Information". Based on the projected future cash flow of the Company for 1989 through 1994 derived from such projections and discount rates (ranging from 9.5% to 13.5%) believed to reflect the Company's weighted average cost of capital, and estimated cash flow multiples of 9 to 11, Lazard estimated that the total value of the Company's assets, less net debt of $970 million (as of March 31, 1989 and including the impact of the acquisition of Lorimar), ranged from approximately $55.55 to $80.13 per share. In addition, the Lazard Presentation contained a discounted present value analysis of the Company's potential stock price in a stand-alone scenario, which also was based in part on the projections provided to Lazard by the Company. Based on estimates of earnings per share derived from such projections and using equity discount rates of 12%, 14% and 16% and price-to-earnings multiples of 16, 18 and 20, Lazard estimated that the net present discounted value of the Company's shares in a stand-alone scenario ranged from approximately $43.75 to $62.99 per share.

(iii) *Research Analyst Reports.* The Lazard Presentation also contained a summary (without an endorsement) of previously published reports prepared by research analysts at ten other nationally recognized investment banking firms which set forth pre-tax break-up analysis valuations of the Company that ranged in value from $53.68 per share to $76.21 per share, with an average value of $62.71 per share.

* * * *

**Fairness Opinions of Time Warner's Financial Advisors**

Each of Wasserstein Perella and Shearson Lehman has delivered a written opinion to the Board of Directors of Time Warner to the effect that, as of June 16, 1989, the financial terms of the acquisition (as set forth in the Merger Agreement in effect on such date) of the Company by Time Warner are fair to Time Warner and its stockholders from a financial point of view.

* * * *

In connection with rendering their fairness opinions and with certain opinions given in connection with the KDS Offer and the Amended KDS Offer, Wasserstein Perella and Shearson Lehman made four presentations to the Board of Directors of Time

---

5. See Appendix D, pages D-2 and D-3.
Warner at meetings held on June 8, 1989, June 11, 1989, June 15-16, 1989, and June 26, 1989 (collectively, the "June Presentations").

* * * *

The information contained in the June Presentations, including the pre-tax segment valuations of the Company and its businesses, was based upon the information reviewed and factors considered by Wasserstein Perella and Shearson Lehman in rendering their respective fairness opinions, discussed above, and does not reflect, among other things, actual financial results of the Company to date, revised prospects for the Company's businesses, changes in general business and economic conditions or any other transaction or event that has occurred or that may occur and that was not anticipated at the time such information was prepared, including the recent sale of a major motion picture studio.

(i) Pre-tax Segment Valuation. Wasserstein Perella and Shearson Lehman performed their pre-tax segment valuation of the Company through discounted cash flow analyses, 6 comparable acquisitions analyses and analyses 7 of the public market trading values and private market values of comparable companies 8 for each of the filmed entertainment, music/music publishing, cable television, publishing and Lorimar units of the Company. 9 Such analyses were based in part on certain estimated financial information for 1989 and 1990 provided to Wasserstein Perella and Shearson Lehman by the Company and on extrapolations of such projections through 1998 performed by Wasserstein Perella and Shearson Lehman. 10 See "—Certain Estimated Financial Information". Based on such projections and analyses, Wasserstein Perella and Shearson Lehman estimated that the Company had a pre-tax value at the time of the June Presentations of between $13.58 billion and $15.31 billion or between $68.40 and $77.12 per share. 11

(ii) Comparable Acquisition Multiples. Wasserstein Perella and Shearson Lehman also reviewed multiples of revenue and operating income in five acquisitions of media companies since November 1987. Multiples of revenues in such acquisitions ranged from 1.3 to 4.0 and averaged 2.56. Multiples of operating income in such acquisitions ranged from 10.0 to 35.8 and averaged 22.13, although such multiple was not available for one acquisition and not material for another. By contrast, a $70 per share acquisition of the Company would represent a multiple of revenues equal to 2.5 and a multiple of operating income equal to 17.8.

6. See Appendix C, pages C-2 and C-3 (regarding DCF and the film segment).
7. See Appendix C, page C-4 (involving comparable transactions analysis and the film segment).
8. See Appendix C, page C-5 (illustrating comparable companies analysis and the film segment).
9. See Appendix C, page C-6 (summarizing pre-tax segment valuationof assets).
10. See Appendix C, page C-2 (regarding the film segment).
11. See Appendix C, page C-7 (summarizing pre-tax segment valuation of equity).
(iii) Research Analyst Reports. At the June 26, 1989 meeting of the Board of Directors of Time Warner, Wasserstein Perella and Shearson Lehman also presented the published comments of two independent research analysts concerning the value of Time Warner after the Merger. Wasserstein Perella and Shearson Lehman discussed the fact that both reports estimated that Time Warner should be valued at $173 per share after the Merger. Each of Wasserstein Perella and Shearson Lehman stated that such estimates were not estimates by Wasserstein Perella or Shearson Lehman of the value of Time Warner after the Merger and were provided for comparative purposes only.

* * * *
Dear Members of the Board:

You have requested our opinion as to the fairness, from a financial point of view, to the stockholders of Warner Communications Inc. ("WCI") of the consideration to be received pursuant to the Agreement and Plan of Merger, dated March 3, 1989, as Amended and Restated as of May 19, 1989 and as Further Amended and Restated as of June 16, 1989 (the "Merger Agreement"), among Time Incorporated ("Time"), TW Sub Inc., a wholly owned subsidiary of Time ("Sub"), and WCI, providing for the acquisition of WCI by Time.

* * * *

In arriving at our opinion, we have, among other things:

* * * *

(vi) analyzed certain historical business and financial information relating to WCI and Time, including the Annual Reports to Stockholders and Annual Reports on Form 10-K of WCI and Time for each of the fiscal years ended December 31, 1985 through 1988, the Quarterly Reports on Form 10-Q of WCI and Time for the quarter ended March 31, 1989, and the Current Report on Form 8-K of WCI, dated March 17, 1989,

(vii) reviewed certain financial forecasts and other data provided to us by WCI and Time relating to the businesses of both companies,

* * * *

Based on the foregoing and upon such other factors as we deemed relevant, including our assessment of current economic, monetary and market conditions, we are of the opinion that as of the date hereof, the Consideration to be received by WCI's stockholders in the Offer and the Merger when taken together is [fair] to WCI's stockholders (other than Time) from a financial point of view.

Very truly yours,
/Lazard Ferès
APPENDIX C

EXcerpts from joint presentation by Wasserstein Perella & Co., Inc. and Shearson Lehman Hutton, Inc. to Time's Board:
Exhibit (b)(5) to Schedule 13E-3
### Appendix C

#### Exhibit C

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<th>Projected</th>
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<tr>
<td>1994</td>
<td>110,000</td>
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#### Unlevered Net Income

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#### Unlevered Cash Flow

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<th>Projected</th>
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#### Assumptions

- Initial investment
- Cash flow
- Capital expenditures
- Leases
- Debt
- Depreciation & amortization
- Revenues from

---

Appendix C
### WONDER - FILMED ENTERTAINMENT
Discounted Cash Flow Analysis

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Present Value of Cash Flow
Present Value of Terminal Cash Flow

### PRESENT VALUE OF SEGMENT

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Present Value of Cash Flow
Present Value of Terminal Cash Flow

### PRESENT VALUE OF SEGMENT

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Present Value of Cash Flow
Present Value of Terminal Cash Flow

### PRESENT VALUE OF SEGMENT
Appendix C

**Wonderland**

**Comparable Transaction Acquisition Analysis**

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<tr>
<th>Year</th>
<th>EBIT</th>
<th>Net Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>272.2</td>
<td>3,571.7</td>
</tr>
<tr>
<td>2006</td>
<td>272.2</td>
<td>3,571.7</td>
</tr>
</tbody>
</table>

**1998 Find**
### WONDER

#### Comparable Company Valuation Analysis

**Filmed Entertainment ($MM)**

<table>
<thead>
<tr>
<th>Unlevered Market Value as a Multiple of LTM</th>
<th># of Co.</th>
<th>1989 Fined Entertainment</th>
<th>Implied Asset Value Range</th>
<th>Acquisition Value (50% Premium)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inc.</td>
<td>Mean</td>
<td>Median</td>
<td>High</td>
</tr>
<tr>
<td>Net Sales</td>
<td>5</td>
<td>2.1</td>
<td>1.8</td>
<td>3.1</td>
</tr>
<tr>
<td>EBIT</td>
<td>4</td>
<td>12.7</td>
<td>13.7</td>
<td>15.0</td>
</tr>
</tbody>
</table>

---

*Motion Picture Industry Comparable Company Analysis*

#### Market Data and Valuation Multiples

<table>
<thead>
<tr>
<th>Ticker Symbol</th>
<th>DIS</th>
<th>GIEF</th>
<th>CI</th>
<th>COLUMBIA PICTURES</th>
<th>MCA INC</th>
<th>MGM</th>
<th>MAMBO (a)</th>
<th>ORION PICTURES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange Traded On</td>
<td>NYSE</td>
<td>GW</td>
<td>NYSE</td>
<td>MCA</td>
<td>MGM</td>
<td>NYSE</td>
<td>OPC</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price Per Share</th>
<th>Current ($9/90)</th>
<th>$94,000</th>
<th>$59,125</th>
<th>$21,250</th>
<th>$56,625</th>
<th>$18,625</th>
<th>$22,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTD Range</td>
<td>95,875</td>
<td>64,875</td>
<td>55,750</td>
<td>39,500</td>
<td>21,500</td>
<td>11,875</td>
<td>58,750</td>
</tr>
<tr>
<td>1989 Range</td>
<td>68,375</td>
<td>54,000</td>
<td>45,750</td>
<td>34,063</td>
<td>12,500</td>
<td>6,750</td>
<td>50,125</td>
</tr>
<tr>
<td>1987 Range</td>
<td>82,500</td>
<td>41,250</td>
<td>46,750</td>
<td>23,375</td>
<td>16,125</td>
<td>7,250</td>
<td>64,500</td>
</tr>
<tr>
<td>1986 Range</td>
<td>54,875</td>
<td>28,003</td>
<td>36,250</td>
<td>23,875</td>
<td>16,125</td>
<td>8,500</td>
<td>56,800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Earnings Per Share</th>
<th>1990 Estimated (d)</th>
<th>5.52</th>
<th>4.00</th>
<th>0.46</th>
<th>2.60</th>
<th>0.25</th>
<th>0.77</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989 Estimated (d)</td>
<td>4.72</td>
<td>3.55</td>
<td>0.20</td>
<td>2.40</td>
<td>0.42</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Latest 12 Months</td>
<td>4.35</td>
<td>2.25</td>
<td>0.14</td>
<td>2.26</td>
<td>2.00</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>3.80</td>
<td>2.31</td>
<td>0.14</td>
<td>2.26</td>
<td>0.97</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>2.85</td>
<td>1.29</td>
<td>0.58</td>
<td>1.82</td>
<td>1.74</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>1.87</td>
<td>0.42</td>
<td>1.36</td>
<td>1.97</td>
<td>0.31</td>
<td>0.17</td>
<td></td>
</tr>
</tbody>
</table>

| 3 Year Compound Growth | 55.58% | 135.08% | -67.63% | 7.11% | N.M. | N.M. |

<table>
<thead>
<tr>
<th>Price-Earnings Ratios:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Price to:</td>
<td></td>
</tr>
<tr>
<td>1990 Estimated EPS</td>
<td>17.03</td>
</tr>
<tr>
<td>1989 Estimated EPS</td>
<td>19.92</td>
</tr>
<tr>
<td>Latest 12 Months EPS</td>
<td>21.61</td>
</tr>
<tr>
<td>Historical Price to:</td>
<td></td>
</tr>
<tr>
<td>1988 EPS</td>
<td>17.99</td>
</tr>
<tr>
<td>1987 EPS</td>
<td>28.95</td>
</tr>
<tr>
<td>1986 EPS</td>
<td>34.95</td>
</tr>
</tbody>
</table>

| Price to Book Value Per Share | 4.77 x | 1.97 x | 2.39 x | 2.42 x | 2.71 x | 2.48 x |

*See part XI.F (explaining this chart).*
<table>
<thead>
<tr>
<th>Value Range</th>
<th>Value</th>
<th>Comparable Companies</th>
<th>Acquisitions</th>
<th>Comparable Companies</th>
<th>DCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200 - 1,400</td>
<td>1,200 - 1,400</td>
<td>1,200 - 1,400</td>
<td>1,200 - 1,400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175 - 225</td>
<td>390</td>
<td>310 - 330</td>
<td>160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5,000 - 5,500</td>
<td>NA - NA</td>
<td>4.250 - 8.630</td>
<td>5.500 - 5.900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.900 - $3.500</td>
<td>$5.250 - $5.150</td>
<td>$5.700 - $2.700</td>
<td>$2.500 - $2.700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**WONDER**
### PRETAX SEGMENT VALUATION

($MM except per share)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Asset Value</th>
<th>Sales</th>
<th>EBITDA</th>
<th>Operating Income (MM)</th>
<th>Sales</th>
<th>EBITDA</th>
<th>Operating Income</th>
<th>Multiple of 1989E</th>
<th>Subscribers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filmed Entertainment</td>
<td>$3,000</td>
<td>$3,500</td>
<td>$1,961.4</td>
<td>1.5</td>
<td>$220.9</td>
<td>1.8</td>
<td>$255.0</td>
<td>13.6</td>
<td>$13.7</td>
</tr>
<tr>
<td>Records</td>
<td>5,500</td>
<td>5,500</td>
<td>2,656.2</td>
<td>1.9</td>
<td>496.0</td>
<td>2.1</td>
<td>425.0</td>
<td>10.1</td>
<td>11.1</td>
</tr>
<tr>
<td>Cable TV (2)</td>
<td>3,705</td>
<td>4,210</td>
<td>577.5</td>
<td>6.4</td>
<td>226.8</td>
<td>7.3</td>
<td>100.7</td>
<td>16.3</td>
<td>12.9</td>
</tr>
<tr>
<td>Publishing</td>
<td>175</td>
<td>200</td>
<td>165.3</td>
<td>1.1</td>
<td>21.8</td>
<td>1.2</td>
<td>16.2</td>
<td>8.0</td>
<td>10.8</td>
</tr>
<tr>
<td>MARS (3)</td>
<td>1,200</td>
<td>1,400</td>
<td>516.3</td>
<td>2.3</td>
<td>39.6</td>
<td>2.7</td>
<td>29.5</td>
<td>30.3</td>
<td>40.7</td>
</tr>
</tbody>
</table>

**Segment Subtotal** $13,080 - $14,810

**Corporate Adjustments**

Add:
- Cash and cash equivalents (4) $256.6
- Option proceeds (5) 207.0
- Cinamericgo Investment (6) 86.6
- Viacom Investment (7) 51.0
- Hasbro Investment (8) 154.0
- Atari Investment (9) 120.7
- Franklin Mint Investment (10) 68.0
- TBS Investment (11) 94.2
- CVN Companies (12) 10.6
- Excess Real Estate (13) 50.0
- Pittsburgh TV Stations Note (14) 32.0
- QUICS Cable (15) 24.0
- Brooklyn/Queens Cable (16) 76.0
- Movietime Investments (17) 4.8
- Venture Capital Investments (18) 50.0
- Misc. Operating Affiliates (19) 10.5
- Overfunded Pension (20) 20.3
- NOL's and ITC's (21) 164.3
- Interest in BHC (22) 400.0

**LOW**  **HIGH**

| EQUITY VALUE | $13,577 | $15,307 |
| Per Share (24) | $68.40 | $77.12 |

**Loss**

- Short-term debt (4) 30.9
- Long-term debt (2)(4) 1,195.5
- Corporate Overhead (23) 163.2

**Total Adjustments** $497.0
APPENDIX D

EXCERPTS FROM THE PRESENTATION TO THE WARNER BOARD BY LAZARD REGARDING THE VALUATION OF WARNER: EXHIBIT (b)(2) TO THE TIME/WARNER SCHEDULE 13E-3
PROJECT CLOCK

Presentation to the Board of Directors

June 16, 1989
<table>
<thead>
<tr>
<th>Earnings per Share (EPS)</th>
<th>Sum of 9% Dividend</th>
<th>Dividend Distribution</th>
<th>Price-Earnings Ratio</th>
<th>Price/Earnings Multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.374</td>
<td>16x</td>
<td>16</td>
<td>16x</td>
<td></td>
</tr>
</tbody>
</table>

*Note: These figures are based on projections and assumptions.*
## PROJECT CLOCK

**Discounted Present Value Analysis of Potential Warner Stock Price**

**Warner Stand-alone Scenario**

1992 Estimated EPS

\$4.32(a)

<table>
<thead>
<tr>
<th>Potential Stock Price Assuming</th>
<th>16x</th>
<th>18x</th>
<th>20x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price/Earnings Multiple of:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$69.12</td>
<td>$77.76</td>
<td>$86.40</td>
</tr>
</tbody>
</table>

### Net Present Value Analysis(b)

<table>
<thead>
<tr>
<th>Trading Range of Common Equity</th>
<th>Discount Rate</th>
<th>16x</th>
<th>18x</th>
<th>20x</th>
<th>Plus:</th>
<th>Total Discounted Value</th>
<th>at June 30, 1989</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.0 %</td>
<td>$46.49</td>
<td>$52.30</td>
<td>$58.11</td>
<td>Dividend Stream(c)</td>
<td>$1.88</td>
<td>$3.00</td>
</tr>
<tr>
<td></td>
<td>14.0</td>
<td>43.70</td>
<td>49.16</td>
<td>54.62</td>
<td>Plus: BHC Distribution(d)</td>
<td>1.82</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>41.12</td>
<td>46.25</td>
<td>51.39</td>
<td></td>
<td>1.76</td>
<td>3.00</td>
</tr>
</tbody>
</table>

(a) From projections provided by Warner.
(b) Present value calculated as of June 30, 1989.
(c) Assumes per share dividend distributed quarterly of \$0.17 per quarter or \$0.68 per annum.
(d) Assumes that the Series B preferred stock, which is convertible into 24 million common shares, receives the BHC distribution.
Appendix D

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>3,069</td>
<td>3,117</td>
<td>3,241</td>
<td>3,230</td>
<td>3,269</td>
<td>3,294</td>
</tr>
<tr>
<td>Depreciation and Amortization</td>
<td>$583</td>
<td>$538</td>
<td>$520</td>
<td>$515</td>
<td>$555</td>
<td>$530</td>
</tr>
<tr>
<td>Depreciation &amp; Disposition, Net</td>
<td>$486</td>
<td>$387</td>
<td>$373</td>
<td>$371</td>
<td>$366</td>
<td>$351</td>
</tr>
<tr>
<td>Net Working Capital Changes</td>
<td>$12</td>
<td>$15</td>
<td>$17</td>
<td>$18</td>
<td>$19</td>
<td>$20</td>
</tr>
<tr>
<td>Capital Expenditures</td>
<td>$27</td>
<td>$28</td>
<td>$29</td>
<td>$30</td>
<td>$31</td>
<td>$32</td>
</tr>
<tr>
<td>Division Income</td>
<td>$699</td>
<td>$597</td>
<td>$586</td>
<td>$581</td>
<td>$555</td>
<td>$530</td>
</tr>
<tr>
<td>Unlevered Net Income</td>
<td>$17</td>
<td>$20</td>
<td>$22</td>
<td>$23</td>
<td>$24</td>
<td>$25</td>
</tr>
<tr>
<td>Tax (q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the Year Ending December 31 (q)

$ in millions, except per share figures.

Consolidated Unlevered DCF Analysis of Warner.
PROJECT CLOCK

Filmed Entertainment Segment*  
($ in millions)

I. Business Description

The filmed entertainment segment includes production, financing and distribution of theatrical films, television programming and home video. From 1989, the segment includes the television programming operations of Lorimar.

II. Summary Financial Data

<table>
<thead>
<tr>
<th></th>
<th>1988 (a)</th>
<th>1989E</th>
<th>1990E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$1,571</td>
<td>$2,400</td>
<td>$3,103</td>
</tr>
<tr>
<td>Operating Cash Flow (EBDIT)</td>
<td>208</td>
<td>299</td>
<td>390</td>
</tr>
<tr>
<td>Operating Income</td>
<td>203</td>
<td>290</td>
<td>375</td>
</tr>
<tr>
<td>Pro Forma Net Income (b)</td>
<td>120</td>
<td>174</td>
<td>225</td>
</tr>
</tbody>
</table>

III. Public Market Range

<table>
<thead>
<tr>
<th></th>
<th>Selected Comparable Public Companies</th>
<th>Implied Value Range Based on 1989E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Mkt. Cap/Revenues</td>
<td>1.5x</td>
<td>3.1x</td>
</tr>
<tr>
<td>Mkt. Cap/EBDIT</td>
<td>9.3</td>
<td>18.1</td>
</tr>
<tr>
<td>Mkt. Cap/Operating Income</td>
<td>10.4</td>
<td>24.0</td>
</tr>
<tr>
<td>Mkt. Value/Net Income</td>
<td>17.8</td>
<td>28.1</td>
</tr>
</tbody>
</table>

* See part XI.F (explaining this chart).

PROJECT CLOCK

Filmed Entertainment Segment*
Appendix D

1996

V. Discounted Cash Flow Range

<table>
<thead>
<tr>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value/Revenues</td>
<td>Acquisition</td>
</tr>
<tr>
<td>6.8x</td>
<td>3.1x</td>
</tr>
<tr>
<td>6.9</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.0</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.1</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.2</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.3</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.4</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.5</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.6</td>
<td>3.1x</td>
</tr>
<tr>
<td>7.7</td>
<td>3.1x</td>
</tr>
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<td>7.8</td>
<td>3.1x</td>
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<tr>
<td>7.9</td>
<td>3.1x</td>
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<td>8.0</td>
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<td>3.1x</td>
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</tr>
<tr>
<td>9.8</td>
<td>3.1x</td>
</tr>
<tr>
<td>9.9</td>
<td>3.1x</td>
</tr>
<tr>
<td>10.0</td>
<td>3.1x</td>
</tr>
</tbody>
</table>

4.186 4.398
4.107
3.877
3.751
3.015

11.0%
13.0%
15.0%

Disyeet Rate

Discounted Cash Flow Multiplier of Net Present Value at

(4) 1988 data excludes the impact of the acquisition of Lottomart.

(3) Assumes a 40% tax rate and no utilization of tax attributes.

See part XI.E (explaining this chart).
APPENDIX E

SOLVENCY OPINION ISSUED BY THE MANUFACTURER’S APPRAISAL COMPANY TO THE LENDERS IN TIME’S ACQUISITION OF WARNER: SCHEDULE (b)(15) TO THE TIME/WARNER SCHEDULE 13E-3

THE MANUFACTURERS’ APPRAISAL COMPANY

The Lenders now or hereafter parties to the Credit Agreement

* * * * *

Ladies and Gentlemen:

Pursuant to an Amended and Restated Agreement and Plan of Merger dated as of June 16, 1989 (the "Merger Agreement"), among Time Incorporated ("Time"), TW Sub Inc. ("the Merger Sub") and Warner Communications Inc. ("Warner"), and the related Offer to Purchase of Time dated June 16, 1989, Time has offered to purchase 100 million outstanding shares of Common Stock, par value $1 per share (the "Shares"), of Warner at $70.00 per share (the "Tender Offer").

* * * *

The amount required to purchase 100 million shares and to pay related fees and expenses is estimated to be $7.3 billion, and the amount required to pay the Merger Consideration will depend on the portion of the Merger Consideration which is cash. At least one half of the Merger Consideration will be comprised of equity securities of Time.

* * * *

Finally, we have assumed for the purposes of this letter that the Credit Agreements will be refinanced on the Merger Date through a new bank facility of facilities in an aggregate principal amount of at least $11.5 billion, having a final maturity of seven years and an average life of approximately four years.

The aforementioned transactions including the Merger and the transaction resulting from the Tender Offer are hereinafter referred to collectively as the “Transactions.” The financing vehicles related to the Transactions are hereinafter referred to collectively as the “Financing.” The merged entity resulting from the effectuation of the Transactions, consisting essentially of Time and Warner and their respective businesses, is hereinafter referred to as the “Company.”
You have requested our opinion as to whether, after giving effect to the Transactions and the Financing:

(i) On a pro forma basis, the fair saleable value of the Company's assets exceeds its stated liabilities, including identified contingent liabilities.
(ii) The Company will be able to pay its debts when due.
(iii) The Company will not have unreasonably small capital with which to conduct its business.

For purposes of this letter, certain terms are defined as follows:

(1) "fair saleable value" is defined as the aggregate amount at which the assets of the Company, valued in its entirety as a going concern, would change hands in the open market, assuming a sale by a willing buyer to a willing purchaser dealing at arm's length, each having reasonable knowledge of all relevant facts, with neither party being under any compulsion to act. We believe that this is a reasonable basis on which to value the Company, and nothing has come to our attention which causes us to believe that the Company both before and after the Transactions and the Financing is not a going concern;

   * * * *

(3) "able to pay its debts when due" means that, assuming the Transactions and the Financing have been consummated as proposed, during the period covered by the financial projections prepared by management of the Company, the cash flow of the Company will be sufficient to provide cash necessary to pay the Company's debts (including those related to contingent liabilities) as such debts mature and become due; and

(4) "will not have unreasonably small capital with which to conduct its business" means that the Company will not lack sufficient capital for the needs (including the anticipated needs) for capital of its businesses, including requirements related to contingent liabilities, as management of the Company has indicated the business of the Company is proposed to be conducted following the consummation of the Transactions, and after giving due consideration to the prevailing practices in the industries in which it will be engaged.

In the course of rendering the opinion contained in this letter, we visited various of the Company's facilities, and interviewed certain Company personnel, including management. Furthermore, we reviewed and considered various data, including but not limited to, the following:

   * * * *

(7) Financial models, including pro forma income statements, balance sheets, cash flow statements, and supporting schedules reflecting the subject Transactions.
While various judgments and estimates which consider reasonable and appropriate under the circumstances were made by us in the determination of fair saleable value, no assurance can be given by us that the sale price which might ultimately be realized in an actual transaction, if and when effected, will be at the fair saleable value estimate resulting from our analysis.

* * * *
APPENDIX F

EXCERPT FROM JOINT BANK AMERICA (BAC) AND SECURITY PACIFIC (SPC) PROXY STATEMENTS AND BANK AMERICA PROSPECTUS, DATED NOVEMBER 14, 1991

OPINIONS OF FINANCIAL ADVISORS

BAC. BAC has retained Morgan Stanley to act as its financial advisor in connection with the Merger. Morgan Stanley delivered to the BAC Board of Directors its written opinion that, based upon and subject to the factors and assumptions set forth in the written opinion, on August 11, 1991 the Conversion Ratio was fair from a financial point of view to BAC. No limitations were imposed by BAC with respect to the investigations made or the procedures followed by Morgan Stanley in rendering its opinion.

The full text of the opinion of Morgan Stanley, which sets forth assumptions made, matters considered and limitations on the review undertaken by Morgan Stanley, is attached as Annex 5 to this Joint Proxy Statement. BAC shareholders are urged to read the opinion in its entirety. Morgan Stanley’s opinion is directed only to the Conversion Ratio and does not constitute a recommendation to any BAC shareholder as to how such shareholder should vote at the BAC Meeting. The summary of the opinion of Morgan Stanley set forth in this Joint Proxy Statement is qualified in its entirety by reference to the opinion.

* * * *

The following is a summary of the analyses Morgan Stanley utilized in arriving at its opinion as to the fairness of the Conversion Ratio and that Morgan Stanley discussed with the BAC Board of Directors on August 10 and 11, 1991.

Overview of SPC. Morgan Stanley evaluated the positions and strengths of SPC in certain lines of business, certain financial and operating information of SPC (including historical net income, nonperforming assets, net charge-off ratios and the breakdown of the loan portfolio), SPC’s announced restructuring plans and the stock market trading history of SPC.

In addition, Morgan Stanley reviewed the historical market to book multiples of SPC and BAC and the relationship between movements of such common stocks and movements of a composite index comprised of comparable bank holding companies.

SPC Valuation Analysis. Morgan Stanley arrived at values for SPC using four valuation methodologies: an analysis of the relationship between return on equity and price to book value ratio; a comparable company analysis; an analysis of prices and terms of recent mergers and acquisitions in-
volving comparable bank holding companies and a dividend discount analy-
sis. These methodologies are discussed below:

**Analysis of Relationship Between Return on Equity and Price to Book Value.** Based on a sampling of approximately 40 publicly traded bank holding companies, Morgan Stanley determined a theoretical relationship in the market between the ratio of expected sustainable return on equity to risk-adjusted required return on investment and price to book value.

* * * *

**Comparable Company Analysis.** Comparable company analysis analyzes a company's operating performance relative to a group of publicly traded peers. Based on relative performance and outlook for a company versus its peers, this analysis enables an implied unaffected market trading value to be determined.

* * * *

**Comparable Merger and Acquisition Analysis.** Comparable merger and acquisition analysis provides a valuation range based upon premiums paid for selected bank holding companies in recent transactions.

* * * *

**Dividend Discount Analysis.** The stand-alone valuation of SPC was estimated by adding (i) the present value of future dividend streams that SPC could produce over a five-year period from 1992 through 1996 and (ii) the present value of SPC's 1996 terminal value. The terminal value of SPC's common equity at the end of the five-year period was determined by applying a range of multiples (from 1.0x to 1.2x) to SPC's terminal year book value. Earnings were projected assuming SPC performed in accordance with forecasted results of operations and certain variations thereof. The dividend streams and terminal values were discounted to present values using a range of discount rates (from 15% to 16%) chosen to reflect different assumptions regarding the required rates of return of holders or prospective buyers of SPC's common equity. The hypothetical maximum value of SPC to BAC was determined by adding the range of stand-alone valuations of SPC (which ranged from approximately $23.50 to approximately $32.50 per share of SPC Common Stock) to the present value of after-tax cost savings, net of merger expenses and deposit-run off (the total of which was assumed to be approximately $28 per share of SPC Common Stock), resulting in a range of $51.50 to $60.50 per
share of SPC Common Stock. This valuation method allocates all of the cost savings to SPC's value, even though SPC could not generate those values as an independent concern. In addition, this valuation does not reflect the fact that on October 15, 1991, SPC announced that its Board of Directors suspended the payment of further cash dividends on shares of SPC Common Stock.